Testing non-linear dependence and market timing ability. An application to the Hedge Fund industry

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The hedge fund industry has experienced a rapid growth in the past 15 years (28% per year, according to HFR).

As of January 2004, Hedge funds assets under management were $817 billion and the estimated number of funds was 6,297 (HFR).

It is widely acknowledged that Hedge funds can be useful in eliminating market inefficiencies and enhancing investment diversification.

However, this recent rapid growth has raised concerns about the characteristics of their risks.

Hence, understanding the risk exposures that affect hedge funds is a crucial issue.
Basic features

- Actively managed pooled investment vehicle
- Performance measured in absolute return units
- Open to a limited group of investors willing to pay hefty inventive fees
- Hedge fund managers exploit their supposed “informational advantages”:
  - Concentrate their positions on a few assets and hedge all other risks
  - eg: buy undervalued/sell overvalued assets (long-short)
  - Often use leverage to magnify the effect
• **Event driven**: equity-oriented investments to capture price movements generated by corporate events (merger, restructuring, liquidation,...)

• **Market neutral**: exploit market inefficiencies investing in long/short equity portfolios

• **Equity hedge**: Equity-oriented investing on both the long and short sides of the market, with an objective different from being market neutral

• **Equity non hedge**: similar to Equity hedge but using predominantly long positions

• **Funds of funds**: diversified portfolios of hedge funds
Other styles

- **Convertible arbitrage**: investments in the convertible securities of a company (e.g. long the convertible bond and short the stock of the same company)

- **Dedicated short bias**: short positions in mostly equities and derivatives

- **Emerging markets**: equity of fixed income investments in emerging markets

- **Fixed income arbitrage**: exploit price anomalies between related interest rate securities

- **Global macro**: long and short positions in capital and derivative markets based on bets on major economic trends or events

- **Managed futures**: investments in listed financial and commodity futures markets and currency markets
## Performance 1994-2005

### CSFB-Tremont style indices

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<th>Mean</th>
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### Annualised statistics

Testing dependence between financial returns
Risks exposures

- Traditional models have typically assumed a linear factor structure to model dependence to market factors \((y_{m1}, \ldots, y_{mk})\)

\[
y = \beta_0 + \beta_1 y_{m1} + \beta_2 y_{m2} + \cdots + \beta_k y_{mk} + \varepsilon
\]

- Linear factor models have not been able to find economically relevant market exposures (Fung and Hsieh, 1997)

- However, nonlinear dependence may still exist:
  - Fung and Hsieh (2001), and Agarwal and Naik (2004): option-like payoffs with respect to the market
  - Lo (2001): “phase-locking” behaviour (e.g. Russian debt restructuring, August 1998)

- Hence, non-linearities can have far more harming effects!
Main contributions of this paper

1. Propose parametric tests of dependence between groups of variables:
   • Tests of linear (Gaussian) and Student $t$ dependence
   • Generalised Hyperbolic (GH) alternative in mind, but also useful against other nonlinear alternatives
   • Nonlinear dependence in both the conditional mean and variance
   • Multivariate perspective
   • Time varying features (e.g. AR, GARCH effects)
Main contributions of this paper

2. Relate those tests to market timing abilities

- Private information about the future state of the economy can generate beneficial nonlinear dependence

- Unfortunately, if this information is biased or just pure noise, its effect on returns can be very harmful
3. Apply those tests to analyse dependence in the hedge fund industry:

- Nonlinear dependence with respect to market (S&P 500, FT100) is found for a significant number of funds.
- Student $t$ dependence provides a much better fit.
- Market mis-timing seems to be more prevalent than market timing.
- Nonlinear dependence seems to improve the diversification benefits of hedge funds in normal times, but deteriorates their performance in bad times.
Plan of the talk

- Effects of market timing abilities on returns
- Relationship with the effects introduced by the Gaussian, Student $t$ and Generalised Hyperbolic distributions
- Propose parametric tests of dependence between groups of variables
- Study the dependence between Hedge Fund returns and the market
- Conclusions and directions for future research
A fund manager invests the capital of the fund in a risk-free asset \( r \) and a set of \( A \) traded risky assets whose net returns are \( y_a \).

There are \( k < A \) market factors in this economy, denoted by \( y_m \), which are either directly traded or can be spanned by \( y_a \).

We will assume joint normality for \( y_a \) and \( y_m \):

\[
\begin{bmatrix}
  y_m \\
  y_a
\end{bmatrix}
\sim N
\begin{bmatrix}
  \begin{pmatrix}
    \mu_m \\
    \mu_a
  \end{pmatrix},
  \begin{pmatrix}
    \Sigma_{mm} & \Sigma'_{am} \\
    \Sigma'_{am} & \Sigma_{aa}
  \end{pmatrix}
\end{bmatrix},
\]

Hence, we can express \( y_a \) in terms of the following linear factor model:

\[
y_a = \mu_a + \Sigma_{am}\Sigma_{mm}^{-1}(y_m - \mu_m) + \varepsilon_{a|m}
\]

where \( \varepsilon_{a|m} \sim N(0, \Sigma_{a|m}) \) is independent of \( y_m \), and

\[
\Sigma_{a|m} = \Sigma_{aa} - \Sigma_{am}\Sigma_{mm}^{-1}\Sigma'_{am}.
\]
Hedge fund manager without market timing abilities

- She maximises a CARA utility

\[ U(w) = -\exp[-\alpha[(1 - w'\lambda_A)r + w'y_a]] \]

- Optimal portfolio weights:

\[ w^0 = \alpha^{-1} \Sigma_{aa}^{-1}(\mu_a - r\lambda_A) \]

- Optimal portfolio:

\[ y^0_h = (1 - w^0'\lambda_A)r + w^0'y_a \]
• Optimal portfolio conditional moments:

\[
E \left[ y_h^0 \mid y_m \right] = r + \alpha^{-1} (\mu_a - r\iota_A)' \Sigma_{aa}^{-1} (\mu_a - r\iota_A) \\
+ \alpha^{-1} (\mu_a - r\iota_A)' \Sigma_{aa}^{-1} \Sigma_{am} \Sigma_{mm}^{-1} (y_m - \mu_m)
\]

\[
V \left[ y_h^0 \mid y_m \right] = \alpha^{-2} (\mu_a - r\iota_A)' \Sigma_{aa}^{-1} \Sigma_{am} \Sigma_{aa}^{-1} (\mu_a - r\iota_A)
\]

or equivalently:

\[
E \left[ y_h \mid y_m \right] = c_0 + c_1'(y_m - \mu_m)
\]

\[
V \left[ y_h \mid y_m \right] = d_0
\]

• The returns of e.g. market neutral funds should reflect this type of dependence, since their managers are not supposed to time the market
Now, the manager is a market timer who receives a signal $s = l(y_m - \mu_m) + \varepsilon_{s|m}$, where $\varepsilon_{s|m} \sim N[0, \lambda \Sigma_{mm}]$ and $\lambda > 0$.

Although she believes that the signal is unbiased ($l = 1$), she might be wrong ($l = -1, l = 0$).

It can be shown that now the conditional mean and variance are

$$E[y_h | y_m] = c_0 + c_1(k)'(y_m - \mu_m) + c_2(k)\varsigma_{mt}$$

$$V[y_h | y_m] = d_0 + d_1'(y_m - \mu_m) + d_2\varsigma_{mt}$$

where

$$\varsigma_{mt} = (y_m - \mu_m)'\Sigma_{mm}^{-1}(y_m - \mu_m)$$

$$c_2(k) = l\alpha^{-1}\lambda^{-1}$$

and the remaining coefficients are positive.
Effects of nonlinear dependence

- Now assume we have data on $N$ alternative assets returns, $y_h$
- We want to analyse its dependence on the $k$ market factors with returns $y_m$
- Mean and Variance
  \[
  E \begin{bmatrix}
  y_m \\
  y_h
  \end{bmatrix} = \begin{bmatrix}
  \mu_m \\
  \mu_h
  \end{bmatrix};
  V \begin{bmatrix}
  y_m \\
  y_h
  \end{bmatrix} = \begin{bmatrix}
  \Sigma_{mm} & \Sigma'_{hm} \\
  \Sigma_{hm} & \Sigma_{hh}
  \end{bmatrix}.
  \]
- We will study the presence of nonlinear dependence and market timing effects by modelling $E (y_h | y_m)$ and $V (y_h | y_m)$
Dependence structures

- Linear or “Gaussian” (No market timing abilities):

\[
E(y_h | y_m) = \mu_h + B(y_m - \mu_m)
\]

\[
V(y_h | y_m) = \Sigma_{h|m} = \Sigma_{hh} - \Sigma_{hm} \Sigma_{mm}^{-1} \Sigma'_{hm}
\]

where \( B = \Sigma_{hm} \Sigma_{mm}^{-1} \)

- Student t dependence (Noisy signal):

\[
E(y_h | y_m) = \mu_h + B(y_m - \mu_m)
\]

\[
V(y_h | y_m) = \frac{1 - 2\eta + \eta \varsigma_{mt}}{1 + \eta(k - 2)} \Sigma_{h|m}
\]

where \( \varsigma_{mt} = (y_m - \mu_m)' \Sigma_{mm}^{-1} (y_m - \mu_m) \)

- Generalised Hyperbolic: nonlinear shape in both the mean and variance, that can account for biased/unbiased market timing abilities
The Generalised Hyperbolic (GH) is a family of multivariate asymmetric and leptokurtic distributions, with the Gaussian and the Student $t$ as particular cases.

In this paper, we will focus on its dependence structure

$$E[y_h|y_m;\eta,\psi,b]; \ V[y_h|y_m;\eta,\psi,b]$$

No further assumptions are made about the marginal distributions.

The parameters of this distribution can generate a wide range of interesting dependence structures:

- $\eta \in (-\infty, \infty), \psi \in [0, 1]$: nonlinear symmetric dependence in the conditional variance
- $b \in \mathbb{R}^{N+k}$: nonlinear asymmetric features in both the conditional mean and variance
Graphical illustration

\[ E(y_h | y_m) \]

\[ \sqrt{V(y_h | y_m)} \]

Type of dependence: Gaussian Student \( t \) GH1 GH2

Market timing signal: None Noisy Biased Unbiased

\[
E \begin{bmatrix} y_m \\ y_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ;
V \begin{bmatrix} y_m \\ y_h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]
Testing linear dependence
Goal

- Testing the null of a linear dependence between $y_{ht}$ and $y_{mt}$
- Correlation is a **sufficient** statistic to “measure” dependence under the null
- Under the null, we can write

$$
\underbrace{y_{ht} - \mu_h}_{\varepsilon_{ht}} = \Sigma_{hm} \Sigma_{mm}^{-1} \underbrace{y_{mt} - \mu_m}_{\varepsilon_{mt}} + \varepsilon_{h|m}
$$

with $\varepsilon_{h|m}$ independent of $\varepsilon_{mt}$

- In particular, we will focus on the implications of the null for the first two moments

$$
E[\varepsilon_{ht}|\varepsilon_{mt}] = \Sigma_{hm} \Sigma_{mm}^{-1} \varepsilon_{mt}; V[\varepsilon_{ht}|\varepsilon_{mt}] = \Sigma_{hh} - \Sigma_{hm} \Sigma_{mm}^{-1} \Sigma_{mh}
$$

- Notation: $\theta = (\mu'_h, \mu'_m, vec'(\Sigma_{hh}), vec'(\Sigma_{hm}), vec'(\Sigma_{mm}))'$
For $k = N = 1$, Patton (2004) considers, for the alternatives

\begin{align*}
E(y_{ht} | y_{mt}) &= \delta_0 + \delta_1 y_{mt} + \delta_2 y_{mt}^2 \\
V(y_{ht} | y_{mt}) &= \omega_0 + \omega_1 y_{mt} + \omega_2 y_{mt}^2
\end{align*}

tests of the following unconditional moments

- Mean neutrality: $\delta_1 = \delta_2 = 0$
- Variance neutrality: $\omega_1 = \omega_2 = 0$
- Linear dependence: $\delta_2 = \omega_1 = \omega_2 = 0$
The approach of this paper

- In this case, we will consider the conditional mean and variance implied by the GH distribution of \( y_{ht} | y_{mt} \) as our alternative.

- Compared to Patton (2004b), the methodology of this paper:
  1. yields the same test in a bivariate unconditional case
  2. can easily account for specific parametric features in the mean and variance (e.g. AR, GARCH effects)
  3. can be reliably implemented in multivariate settings
To test these moment conditions, a sequential methodology will be followed

1. Estimate mean and variance parameters ($\theta$) by Gaussian Pseudo-maximum likelihood (PML): $\hat{\theta}_T$
   Consistent in the absence of normality if mean and variance are correctly specified (Bollerslev and Wooldridge, 1992)

\[
\hat{\theta}_T = \arg \max_{\theta} \frac{1}{T} \sum_{t=1}^{T} \ell_t (y_t; \theta)
\]

where

\[
\ell_t (y_t; \theta) = -\frac{N}{2} \log (2\pi) - \frac{1}{2} \log |\Sigma(\theta)| - \frac{1}{2} [y_t - \mu(\theta)]' \Sigma^{-1}(\theta) [y_t - \mu(\theta)]
\]
2. Consider the Gaussian Pseudo-Likelihood of $y_{ht} | y_{mt}$

$$\ell_{h|mt}(\theta, \eta, \psi, b) = -\frac{N}{2} \log(2\pi) - \frac{1}{2} \log |V [\varepsilon_{ht} | \varepsilon_{mt}; \theta, \eta, \psi, b]|$$

$$- \frac{1}{2} \{\varepsilon_{ht} - E[\varepsilon_{ht} | \varepsilon_{mt}; \theta, \eta, \psi, b]\}'$$

$$\times V^{-1}[\varepsilon_{ht} | \varepsilon_{mt}; \theta, \eta, \psi, b]\{\varepsilon_{ht} - E[\varepsilon_{ht} | \varepsilon_{mt}; \theta, \eta, \psi, b]\}$$

3. Using the first-step parameter estimates $\theta = \hat{\theta}_T$, compute a Lagrange multiplier test of the null of “linearity”

$$E(\varepsilon_{ht} | \varepsilon_{mt}; \theta_0, \eta_0, \psi_0, b_0) = \Sigma_{hm}(\theta_0)\Sigma_{mm}^{-1}(\theta_0)\varepsilon_{mt}(\theta_0)$$

True values

$$V[\varepsilon_{ht}(\theta_0) | \varepsilon_{mt}(\theta_0); \theta_0, \eta_0, \psi_0, b_0] = \Sigma_{h|m}(\theta_0)$$
Intuition

Bivariate case: $k = N = 1$

• Moment conditions in the “linear” dependence test

\[
E \begin{bmatrix}
\text{cov}(\varepsilon_{h \mid mt}^2, \varepsilon_{mt}^2) \\
\text{cov}(\varepsilon_{h \mid mt}^2, \varepsilon_{mt}^*) \\
\text{cov}(\varepsilon_{h \mid mt}^*, \varepsilon_{mt}^2) \\
\text{cov}(\varepsilon_{h \mid mt}^*, \varepsilon_{mt}^*)
\end{bmatrix} = 0
\]

• Moment conditions in the normality test (Mencía and Sentana, 2004)

\[
E \begin{bmatrix}
\sqrt{3/2} \left[ E(\varepsilon_{h \mid mt}^4 - 3) + E(\varepsilon_{mt}^4 - 3) \right] + \text{cov}(\varepsilon_{h \mid mt}^2, \varepsilon_{mt}^2) \\
\sqrt{6} E(\varepsilon_{mt}^3) + \sqrt{2} \text{cov}(\varepsilon_{mt}^*, \varepsilon_{h \mid mt}^2) \\
\sqrt{6} E(\varepsilon_{h \mid mt}^3) + \sqrt{2} \text{cov}(\varepsilon_{h \mid mt}^*, \varepsilon_{mt}^2)
\end{bmatrix} = 0
\]

where $\varepsilon_{m}^* = (y_m - \mu_m)/\sigma_m$, 
$\varepsilon_{h \mid m}^* = [y_h - \mu_h - \sigma_{hm} \sigma_{mm}^{-1}(y_m - \mu_m)]/\sigma_{h \mid m}^{1/2}$

• Moment conditions also relevant for non-GH alternatives
Linear dependence test

Proposition 1

\[
\sup_b \tau_d (\hat{\theta}_T, b) = \left[ \frac{\sqrt{T}}{T} \sum_t \nu_{\eta t} \left( y_{ht} \mid y_{mt}; \hat{\theta}_T \right) \right] \\
\times \mathcal{F}^{-1} (\theta_0) \left[ \frac{\sqrt{T}}{T} \sum_t \nu_{\eta t} \left( y_{ht} \mid y_{mt}; \hat{\theta}_T \right) \right] \to \chi^2_{N+k+1}
\]

where

\[
\nu_{\eta t} (y_{ht} \mid y_{mt}; \theta) = \begin{bmatrix} \frac{1}{2} [s_{h \mid mt} - N] [s_{mt} - k] \\
[s_{h \mid mt} - N] \epsilon_{mt} \\
[s_{mt} - k] \epsilon_{h \mid mt} \end{bmatrix},
\]

\[
s_{mt} = \epsilon'_{mt} \Sigma^{-1}_{mm} \epsilon_{mt}, \]

\[
s_{h \mid mt} = \epsilon'_{h \mid mt} \Sigma^{-1}_{h \mid mt} \epsilon_{h \mid mt},\]

\[
\epsilon_{h \mid mt} = \epsilon_{ht} - \Sigma_{hmt} \Sigma^{-1}_{mmt} \epsilon_{mt}.
\]
Proposition 2

\[ \mathcal{F} (\theta_0) = \mathcal{B}_{\eta \eta} (\theta_0) + \mathcal{A}'_{\theta \eta} (\theta_0) \mathcal{A}^{-1}_{\theta \theta} (\theta_0) \mathcal{B}_{\theta \theta} (\theta_0) \mathcal{A}^{-1}_{\theta \theta} (\theta_0) \mathcal{A}_{\theta \eta} (\theta_0) \]

where \( \mathcal{B}_{\eta \eta} (\theta_0) \) and \( \mathcal{B}_{\theta \theta} (\theta_0) \) are the asymptotic covariance matrices of the PML scores with respect to \( \eta \) and \( \theta \), respectively, and

\[ \mathcal{A}_{\theta \theta} (\theta) = \frac{\partial \mu'(\theta)}{\partial \theta} \Sigma^{-1}(\theta) \frac{\partial \mu(\theta)}{\partial \theta} + \frac{1}{2} \frac{\partial vec' \Sigma(\theta)}{\partial \theta} \left[ \Sigma^{-1}(\theta) \otimes \Sigma^{-1}(\theta) \right] \frac{\partial vec \Sigma(\theta)}{\partial \theta}, \]

\[ \mathcal{A}_{\theta \eta} (\theta) = \left[ \frac{\partial vec' \Sigma_{hm}(\theta)}{\partial \theta} - \frac{\partial vec' \Sigma_{mm}(\theta)}{\partial \theta} \left[ I_k \otimes \Sigma^{-1}_{mm}(\theta) \Sigma_{mh}(\theta) \right] \right] \times \left[ \Sigma^{-1}_{mm}(\theta) \otimes I_N \right] \times \left[ E(s_{mt}(\theta) \varepsilon_{mt}(\theta)) \otimes I_N \right] L'_h, \]

and \( L'_h = [0_N, 0_{Nk}, I_N] \).
Locally equivalent dependence structure

- In the multivariate case, the same test could be obtained from the alternative hypothesis

\[
E(y_{ht} | y_{mt}) = \mu_h + \Sigma_{hm} \Sigma^{-1}_{mm} (y_{mt} - \mu_m) + \delta_2 (\varsigma_{mt} - k)
\]

\[
V(y_{ht} | y_{mt}) = \Sigma_{h|m} [1 + \omega_1' (y_{mt} - \mu_m) + \omega_2 (\varsigma_{mt} - k)]
\]

where \( \varsigma_{mt} = (y_{mt} - \mu_m)' \Sigma^{-1}_{mmt} (y_{mt} - \mu_m) \)

- Null hypothesis: \( \delta_2 = 0, \omega_1 = 0, \omega_2 = 0 \)

- Alternative hypotheses:
  - Nonlinear dependence: \( \delta_2 \neq 0, \omega_1 \neq 0, \omega_2 \neq 0 \)
  - Unbiased signal (ability): \( \delta_2 > 0, \omega_1 > 0, \omega_2 > 0 \)
  - Biased signal (inability): \( \delta_2 < 0, \omega_1 > 0, \omega_2 > 0 \)
  - Noisy signal: \( \delta_2 = 0, \omega_1 > 0, \omega_2 > 0 \)

- By considering one-sided alternatives, we can directly test against market timing effects.
Extensions

- Joint test of Gaussian dependence and absence of correlation
- One-sided test against alternatives with positive co-kurtosis
- Power of the tests and finite sample properties
Testing Student $t$ dependence
• Now the null refers to the conditional mean and variance of a Student $t$ distribution

• In terms of the parameters of the Generalised Hyperbolic distribution, this case corresponds to

$$b = 0; \quad \psi = 1; \quad \eta > 0$$

• Conditional moments under the null:

$$E \left( y_{ht} \mid y_{mt}; \theta \right) = \mu_h(\theta) + \Sigma_{hm}(\theta) \Sigma^{-1}_{mm}(\theta) [y_{mt} - \mu_m(\theta)]$$

$$V \left( y_{ht} \mid y_{mt}; \theta \right) = \frac{1 - 2\eta + \eta s_{mt}(\theta)}{1 - \eta (2 - k)} \Sigma_{h|m}(\theta)$$

where $s_{mt}(\theta) = [y_{mt} - \mu_m(\theta)] \Sigma^{-1}_{mm}(\theta) [y_{mt} - \mu_m(\theta)]$
Estimation procedure

- As in the linearity case, $\theta$ will be estimated by Gaussian PML
- Now there is one additional parameter to estimate, $\eta$
- We can obtain a consistent estimator $\hat{\eta}_T$ from

$$\hat{\eta}_T = \arg\max_{\eta} \left\{ \frac{1}{T} \sum_{t=1}^{T} \ell_{h|mt}(\hat{\theta}_T, \eta, \psi, b) \right\} \quad \text{s.t. } \psi = 1, b = 0$$

- FOC

$$E[s_{nt}(y_{ht}|y_{mt}; \pi_0)] = E \left\{ \frac{s_{mt}(\theta) - k}{2 \left[ 1 - 2\eta + \eta s_{mt}(\theta) \right] \left[ 1 + \eta(k - 2) \right] \left[ 1 - 2\eta + \eta s_{mt}(\theta) \right] - N} \right\} = 0$$
Moment conditions of the test

- Ideally, we would like to use the score of $\ell_{h|mt}(\hat{\theta}_T, \eta, \psi, b)$ with respect to $b$ and $\psi$, where

$$s_{bt} \left( y_{ht} \mid y_{mt}; \pi \right) = \eta \left[ \frac{[1+\eta(k-2)]s_{h|mt}(\theta)}{1-2\eta+\eta s_{mt}(\theta)} - N \right] \frac{\varepsilon_{mt}(\theta)}{1 - 2\eta + \eta s_{mt}(\theta)}$$

- Unfortunately, the score with respect to $\psi$ is identically zero!
Moment conditions of the test

- In such cases, Lee and Chesher (1986) propose an “Extremum test”, which tests the restrictions that the null imposes on higher order conditions.

- In our case, we will use a moment test based on the second order derivative of the pseudo log-likelihood with respect to $\psi$:

$$s_{\psi \psi t}(y_{ht} \mid y_{mt}; \pi) = \eta^2 \left[ \frac{[1 + \eta(k - 2)] \varsigma_{h|mt}(\theta)}{1 - 2\eta + \eta \varsigma_{mt}(\theta)} - N \right]$$

$$\times \left[ \frac{(1 - 4\eta)^{-1}}{1 - 2\eta + \eta \varsigma_{mt}(\theta)} - \frac{(1 - 2\eta)^{-1} [1 - 2\eta + \eta \varsigma_{mt}(\theta)]}{[1 + \eta(k - 2)] [1 + \eta(k - 4)]} \right]$$
Proposition 3

\[ \frac{\sqrt{T}}{T} \sum_t \begin{bmatrix} s_{bt} (y_{ht} | y_{mt}; \hat{\pi}_T) \\ s_{\psi \psi t} (y_{ht} | y_{mt}; \hat{\pi}_T) \end{bmatrix} \xrightarrow{d} N \left[ 0, V (\pi_0) \right] \]

where

\[ V (\pi_0) = \mathcal{J} (\pi_0) \lim_{T \to \infty} V \left\{ \frac{\sqrt{T}}{T} \sum_t \begin{bmatrix} s_{\theta t} (y_t; \theta_0) \\ s_{\eta t} (y_{ht} | y_{mt}; \pi_0) \\ s_{bt} (y_{ht} | y_{mt}; \pi_0) \\ s_{\psi \psi t} (y_{ht} | y_{mt}; \pi_0) \end{bmatrix} \right\} \mathcal{J}' (\pi_0) \]

and \( \mathcal{J} (\pi_0) = \begin{bmatrix} \mathcal{J}_\pi (\pi_0) & I_{N+k+1} \end{bmatrix} \)
Student \( t \) dependence test

\[
\mathcal{J}_\pi (\pi_0) = p \lim_{T \to \infty} \left\{ \frac{-1}{T} \sum_t \frac{\partial}{\partial \pi'} \left[ \begin{array}{c}
 s_{bt} (y_{ht} | y_{mt}; \hat{\pi}_T) \\
 s_{\psi \psi t} (y_{ht} | y_{mt}; \hat{\pi}_T)
\end{array} \right] \right\}
\]

\[
\times \left[ \begin{array}{cc}
 A_{\theta \theta}^{-1} (\theta_0) & 0 \\
 -\mathcal{T}_\eta^{-1} (\pi_0) \mathcal{T}_\theta' (\pi_0) A_{\theta \theta} (\theta_0) & \mathcal{T}_\eta^{-1} (\pi_0)
\end{array} \right]
\]

Hence, our proposed test is

\[
\tau_{2s} (\hat{\pi}_T) = \left\{ \frac{\sqrt{T}}{T} \sum_t \left[ \begin{array}{c}
 s_{bt} (y_{ht} | y_{mt}; \hat{\pi}_T) \\
 s_{\psi \psi t} (y_{ht} | y_{mt}; \hat{\pi}_T)
\end{array} \right] \right\}^\prime \mathcal{V}^{-1} (\hat{\pi}_T)
\]

\[
\times \left\{ \frac{\sqrt{T}}{T} \sum_t \left[ \begin{array}{c}
 s_{bt} (y_{ht} | y_{mt}; \hat{\pi}_T) \\
 s_{\psi \psi t} (y_{ht} | y_{mt}; \hat{\pi}_T)
\end{array} \right] \right\} \to \chi^2_{N+k+1}
\]

Javier Mencía

Testing dependence between financial returns
Empirical application
Dependence in the hedge fund industry

- Hedge funds typically have explicit mandates to make an absolute return target, regardless of the market environment.
- This goal induces most managers to hedge the risk of their positions.
- It is crucial to determine the degree of immunisation that they achieve, in order to assess the quality of those investments.
  - If there is nonlinear dependence, sophisticated dynamic strategies may be necessary to fully hedge market risk.
- We apply the tests derived in the paper to study the dependence between this industry and the market.
Data

- Individual data for around 1,500 hedge funds
  - Monthly returns from 1992 to 2003 (HFR-TASS)
  - Five groups of styles: Event driven, market neutral, equity hedge, equity non-hedge and funds of funds.

- Index data from the CSFB-Tremont hedge fund main style indices
  - Monthly returns from 1994 to 2005
  - Styles: Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Managed Futures and Multi-Strategy

- Market: S&P 500 and FT100 monthly returns
• We will compute the tests developed in the paper to study the dependence between each Hedge fund and the market

\[
\begin{bmatrix}
    y_{mt} \\
    y_{ht}
\end{bmatrix}
= \begin{bmatrix}
    \alpha_m \\
    \alpha_h
\end{bmatrix}
+ \begin{bmatrix}
    \phi_{mm} & 0 \\
    \phi_{mh} & \phi_{hh}
\end{bmatrix}
\begin{bmatrix}
    y_{mt-1} \\
    y_{ht-1}
\end{bmatrix}
+ \Sigma_t^{1/2} \varepsilon_t^*
\]

• Lack of liquidity may cause time series autocorrelation in hedge fund data (Lo, 2001)

• It may also introduce a lagged response of hedge funds to market moves. We allow for this feedback effect when \( \phi_{mh} \neq 0 \).
### Table 1a: Tests with respect to the S&P500

<table>
<thead>
<tr>
<th>Hedge fund strategies</th>
<th>N</th>
<th>Zero correlation</th>
<th>Linear dependence</th>
<th>Joint test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T \geq 30$</td>
<td>(1)</td>
<td>(2)</td>
<td>(1) $\cup$ (2)</td>
</tr>
<tr>
<td>Fund of Funds</td>
<td>404</td>
<td>55.9</td>
<td>23.8</td>
<td>56.7</td>
</tr>
<tr>
<td>Event driven</td>
<td>84</td>
<td>79.8</td>
<td>34.5</td>
<td>71.4</td>
</tr>
<tr>
<td>Market Neutral</td>
<td>92</td>
<td>35.9</td>
<td>14.1</td>
<td>37.0</td>
</tr>
<tr>
<td>Equity non Hedge</td>
<td>79</td>
<td>87.3</td>
<td>10.1</td>
<td>82.3</td>
</tr>
<tr>
<td>Equity Hedge</td>
<td>412</td>
<td>64.8</td>
<td>14.1</td>
<td>62.6</td>
</tr>
</tbody>
</table>

### Table 1b: Tests with respect to the S&P500 and FT100

<table>
<thead>
<tr>
<th>Hedge fund strategies</th>
<th>N</th>
<th>Zero correlation</th>
<th>Linear dependence</th>
<th>Joint test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T \geq 50$</td>
<td>(1)</td>
<td>(2)</td>
<td>(1) $\cup$ (2)</td>
</tr>
<tr>
<td>Fund of Funds</td>
<td>275</td>
<td>62.5</td>
<td>20.4</td>
<td>66.2</td>
</tr>
<tr>
<td>Event driven</td>
<td>67</td>
<td>79.1</td>
<td>29.9</td>
<td>74.6</td>
</tr>
<tr>
<td>Market Neutral</td>
<td>69</td>
<td>27.5</td>
<td>5.8</td>
<td>30.4</td>
</tr>
<tr>
<td>Equity non Hedge</td>
<td>68</td>
<td>86.8</td>
<td>10.3</td>
<td>89.7</td>
</tr>
<tr>
<td>Equity Hedge</td>
<td>279</td>
<td>69.5</td>
<td>10.4</td>
<td>64.5</td>
</tr>
</tbody>
</table>

% of test rejections at the 5% level

Testing dependence between financial returns
Linear dependence vs. Nonlinear dependence and Market timing effects

Tests with respect to the S&P500 ($T \geq 30$)

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Nonlinear</th>
<th>Ability</th>
<th>Inability</th>
<th>Noisy signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund of Funds</td>
<td>23.8</td>
<td>20.8</td>
<td>30.7</td>
<td>24.5</td>
</tr>
<tr>
<td>Event driven</td>
<td>34.5</td>
<td>28.6</td>
<td>38.1</td>
<td>33.3</td>
</tr>
<tr>
<td>Market Neutral</td>
<td>14.1</td>
<td>19.6</td>
<td>19.6</td>
<td>21.7</td>
</tr>
<tr>
<td>Equity non Hedge</td>
<td>10.1</td>
<td>11.4</td>
<td>16.5</td>
<td>12.7</td>
</tr>
<tr>
<td>Equity Hedge</td>
<td>14.1</td>
<td>13.8</td>
<td>17.7</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Tests with respect to the S&P500 and FT100 ($T \geq 50$)

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Nonlinear</th>
<th>Ability</th>
<th>Inability</th>
<th>Noisy signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund of Funds</td>
<td>20.4</td>
<td>24.4</td>
<td>28.0</td>
<td>25.8</td>
</tr>
<tr>
<td>Event driven</td>
<td>29.9</td>
<td>29.9</td>
<td>38.8</td>
<td>32.8</td>
</tr>
<tr>
<td>Market Neutral</td>
<td>5.8</td>
<td>8.7</td>
<td>13.0</td>
<td>11.6</td>
</tr>
<tr>
<td>Equity non Hedge</td>
<td>10.3</td>
<td>11.8</td>
<td>16.2</td>
<td>13.2</td>
</tr>
<tr>
<td>Equity Hedge</td>
<td>10.4</td>
<td>12.5</td>
<td>14.0</td>
<td>14.3</td>
</tr>
</tbody>
</table>

% of test rejections at the 5% level
• We will model the indices and the market with

\[
\begin{bmatrix}
y_{mt} \\
y_{h1t} \\
\vdots \\
y_{hNt}
\end{bmatrix} = \begin{bmatrix}
\alpha_m \\
\alpha_{h1} \\
\vdots \\
\alpha_{hN}
\end{bmatrix} + \begin{bmatrix}
\phi_{mm} & 0 & \cdots & 0 \\
\phi_{mh1} & \phi_{hh1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{mhN} & 0 & \cdots & \phi_{hhN}
\end{bmatrix} \begin{bmatrix}
y_{mt-1} \\
y_{h1t-1} \\
\vdots \\
y_{hNt-1}
\end{bmatrix} + \sum_{t}^{1/2} \mathbf{\varepsilon}_t^*
\]

• Lack of liquidity may cause time series autocorrelation and a lagged response to market moves (feedback) in hedge fund returns (Lo, 2001; Asness, Krail and Liew, 2001)
Joint dependence tests for the 10 CSFB-Tremont main style indices

Triangular VAR(1)-CCC-GARCH(1,1) with feedback

Table 4: Dependence with respect to the S&P 500

<table>
<thead>
<tr>
<th>Null</th>
<th>GH Alternative</th>
<th>Test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian (linear)</td>
<td>2-sided</td>
<td>49.570 (0.000)</td>
</tr>
<tr>
<td>Student t</td>
<td>2-sided</td>
<td>6.579 (0.858)</td>
</tr>
<tr>
<td>Ability</td>
<td></td>
<td>3.601 (0.623)</td>
</tr>
<tr>
<td>Inability</td>
<td></td>
<td>5.576 (0.376)</td>
</tr>
</tbody>
</table>

Table 5: Dependence with respect to the S&P 500 and FT 100

<table>
<thead>
<tr>
<th>Null</th>
<th>GH Alternative</th>
<th>Test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian (linear)</td>
<td>2-sided</td>
<td>30.874 (0.003)</td>
</tr>
<tr>
<td>Student t</td>
<td>2-sided</td>
<td>16.128 (0.214)</td>
</tr>
<tr>
<td>Ability</td>
<td></td>
<td>7.387 (0.275)</td>
</tr>
<tr>
<td>Inability</td>
<td></td>
<td>10.154 (0.134)</td>
</tr>
</tbody>
</table>
Is nonlinear dependence economically relevant for hedge fund investors?

- We can address this issue by comparing the Gaussian dependence structure with the more general nonlinear models.

- Graphically, this comparison can be done with the mean-variance frontiers of the hedged (uncorrelated) portfolio

\[ y_{ht+1}^- = y_{ht+1} - B_t y_{mt}, \]

conditional on information available at \( t \) and \( y_{mt+1} \) on a quartile of its empirical distribution.

- This procedure summarises in a \( 2 - D \) graph the information on the dependence structure given by each model.
We consider four different scenarios for the market return $y_{mt+1}$:

- They are constructed as representative deviations of the market from its expected return:

Empirical distribution of the market’s historical standardised shocks (1994-2005)

\[
\frac{y_{mt+1} - E_t(y_{mt+1})}{V_t^{1/2}(y_{mt+1})} < 0 \quad 0 \quad \frac{y_{mt+1} - E_t(y_{mt+1})}{V_t^{1/2}(y_{mt+1})} > 0
\]
• Conditional mean and variance

\[ E_t \left[ y_{ht+1} \mid y_{mt+1} \in (a, b) \right] = E_t \left[ E_t \left[ y_{ht+1} \mid y_{mt+1} \right] \right] \mid y_{mt+1} \in (a, b) \]

\[ V_t \left[ y_{ht+1} \mid y_{mt+1} \in (a, b) \right] = E_t \left[ V_t \left[ y_{ht+1} \mid y_{mt+1} \right] \right] \mid y_{mt+1} \in (a, b) \]

\[ + \quad V_t \left[ E_t \left[ y_{ht+1} \mid y_{mt+1} \right] \right] \mid y_{mt+1} \in (a, b) \]

• Prospects for the market return at \( t + 1 \)

\[ \tilde{y}_{mt+1,s} = E_t(y_{mt+1}) + V_t^{1/2}(y_{mt+1})\varepsilon^*_s, \quad s = 1, \ldots, T \]

where \( \varepsilon^*_s \) are the historical market standardised returns:

\[ \varepsilon^*_s = \frac{y_{ms} - E_{s-1}(y_{ms})}{V_{s-1}^{1/2}(y_{ms})}, \quad s = 1, \ldots, T \]
Mean-variance frontier of $y_{h_{t+1}}^\perp | y_{m_{t+1}} \in i^{th}$ quartile in August 1998

Very bad scenario

Bad scenario

Good scenario

Very Good scenario

Javier Mencía

Testing dependence between financial returns
Mean-variance frontier of $y_{h_{t+1}|y_{m_{t+1}} \in i^{th} \text{ quartile in August 1998}}$

Using S&P 500 data from 1955-2005 to estimate the moments

Javier Mencía
Conclusions

- We develop tests of Gaussian and Student $t$ dependence
- The tests are only subject to correct specification of the conditional mean and variance, without further assumptions about the marginal distributions
- Dependence between Hedge Fund returns and the market:
  - Nonlinear dependence is important for a significant number of Hedge Funds
  - Noisy signals and even market mis-timing is more prevalent than market timing abilities
  - Joint tests of linear dependence between the whole hedge fund industry and the market are easily rejected
  - The diversification benefits offered by hedge funds are shown to deteriorate in good, and especially in bad times, due to nonlinear dependence
Directions for future research

- Explore the effect of nonlinear dependence on optimal asset allocation
- Study whether hedge funds with higher nonlinear dependence with respect to the market are rewarded with a higher risk-premium
- Source of nonlinear dependence: what proportion is due to hedge funds investment strategies, and what amount did already exist in the primitive assets (Longin and Solnik, 2001; Ang and Chen, 2002)?