
Alejandro Balbás\textsuperscript{1} and José Garrido\textsuperscript{1,2}

\textsuperscript{1}Department of Business Administration
University Carlos III of Madrid, Spain
\textsuperscript{2}Department of Mathematics and Statistics
Concordia University, Canada

Instituto MEFF
April 24, 2002
Overview:

1. Introduction:
   1.1 Distortion Operators, Insurance Pricing,
   1.2 Choquet Pricing of Assets and Losses.

2. Wang’s Distortion Operator,

3. Applications:
   3.1 Asset Pricing,
   3.2 Capital Asset Pricing Model,
   3.3 Black-Scholes Formula,
   3.4 Deviance and Tail Index.

1. Introduction

Insurance and financial risks are becoming more integrated.


The price of an insurance risk (excluding other expenses) is called a risk-adjusted premium. From classical expected utility theory [Borch (1961), Bühlmann (1980), Goovaerts et al. (1984)] a dual theory of risk has emerged in the economic literature [Yaari (1987)].

Similarly, the price of a financial instrument (e.g. options) also carries risk loads. But the markets and the strategies by which insurance and financial risks are sold differ substantially.
We will discuss Wang's proposal of a risk-adjustment method that distorts the survival function of an insurance risk.

Wang (2000) shows that his (one-parameter) distortion operator reproduces Black-Scholes theory for option pricing and extends the Capital Asset Pricing Model (CAPM) to insurance pricing.


We characterize Wang’s transform as a “coherent measure” and propose a dynamic (stochastic process) version. Also a tail index for (financial or insurance) risk distributions is derived from it.
1.1 Distortion Operators in Insurance Pricing.

$X$ is a non-negative loss variable, distributed as $F_X$ and with $S_X = 1 - F_X$ as its survival function.

The net insurance premium (excluding other expenses) is

$$E[X] = \int_0^\infty y dF_X(y) = \int_0^\infty S_X(y) dy.$$  

An insurance layer $X_{(a,a+m]}$ of $X$ is defined by the payoff function

$$X_{(a,a+m]} = \begin{cases} 
0 & 0 \leq X < a \\
X - a & a \leq X < a + m \\
m & a + m \leq X
\end{cases},$$

where $a$ is a deductible and $m$ a payment limit.
The survival function of this insurance layer is given by $S_X$ as

$$S_{X(a,a+m)}(y) = \begin{cases} S_X(a + y) & 0 \leq y < m \\ 0 & m \leq y \end{cases}.$$ 

It yields a net premium of

$$E[X_{(a,a+m)}] = \int_0^\infty S_{X(a,a+m)}(y)dy = \int_a^{a+m} S_X(x)dx.$$ 

Wang (1996) suggested to introduce the risk loading by first distorting the survival function before taking expectations.
Distorting the survival function of any risk, Wang obtains a risk-adjusted premium:

\[ H_g[X] = \int_0^\infty g[S_X(x)]dx, \]

where \( g : [0, 1] \rightarrow [0, 1] \) is increasing with

\[ g(0) = 0 \quad \text{and} \quad g(1) = 1. \]

\( g \) is a distortion operator. It transforms \( S_X \) into a new survival function \( g \circ S_X \), of the “ground-up distribution”.

For example, the risk adjusted premium of a risk layer is then

\[ H_g[X_{(a,a+m)}] = \int_0^\infty g[S_{X_{(a,a+m)}}]dy = \int_a^{a+m} g[S_X(x)]dx. \]
For general insurance pricing, $g$ should satisfy:

- $0 < g(u) < 1$, $g(0) = 0$ and $g(1) = 1$,
- $g$ increasing and concave,
- $g'(0) = +\infty$, where it is defined.

Wang (1996) shows that in a class of one-parameter functions, only

$$g(u) = u^r, \quad u \in [0, 1], \quad 0 < r \leq 1,$$

satisfies all of the above.

This corresponds to the proportional hazards (PH) transform of Wang (1995).
Despite some desirable properties, Wang’s PH transform suffers some major drawbacks. The PH transform:

- of a lognormal is not lognormal (Black-Scholes formula for option pricing),
- lacks flexibility yielding fast increasing risk loadings for high layers,
- cannot be applied simultaneously to assets and liabilities. It yields to serious inconsistencies; see the following section.
1.2 Choquet Pricing of Assets and Losses

Consider an asset $A$ as a negative loss $X = -A$. The Choquet integral with respect to the distortion $g$ is then given by:

$$H_g[X] = \int_{-\infty}^{0} \{g[S_X(x)] - 1\}dx + \int_{0}^{\infty} g[S_X(x)]dx.$$

It has been proposed as a general pricing method for financial markets with frictions [Chateauneuf et al. (1996)].

**Definition:** For any risk $X$ and a real valued $h$, the payoff $Y = h(X)$ is a derivative of $X$. If $h$ is non-decreasing, then $Y$ is called a *comonotone* derivative of the underlying $X$. 


**Theorem:** Under the Choquet integral the price of a comonotone derivative \( Y = h(X) \),

\[
H_g[Y] = \int_{-\infty}^{0} \{g[S_Y(y)] - 1\}dy + \int_{0}^{\infty} g[S_Y(y)]dy,
\]
is equivalent to the expectation of \( h(X_{\alpha}) \), where \( X_{\alpha} \) has the ground-up distribution with survival function \( S_{X_{\alpha}} = g \circ S_X \).

The Choquet integral \( H_g \) provides a “risk-neutral” valuation of comonotone derivatives, and hence of insurance risk layers.

But the above equivalence does not hold for derivatives which are not comonotone with the underlying risk; a distortion with a concave \( g \) always produces non-negative loadings, while the ground-up distribution can yield negative loadings.
Denneberg (1994) shows that for an asset $A > 0$

$$H_g[-A] = -H_g^*[A],$$

where $g^*(u) = 1 - g(1 - u)$ is the dual distortion operator of $g$.

A loss $X$ distorted by a concave $g$ always yields $H_g[X] \geq E[X]$. When $g$ is concave, $g^*$ is convex and $H_g^*[A] \leq E[A]$.

For most choices $g$, the dual $g^*$ belongs to a different parametric family. In particular, if the PH transform $g(u) = u^r$ is applied to losses, the dual $g^*(u) = 1 - (1 - u)^r$ would apply to assets.

Symmetric treatment of assets and liabilities is possible for a new class of distortion operators.
2. Wang’s Distortion Operator

Wang (2000) suggests a new distortion operator defined as:

\[ g_\alpha(u) = \Phi[\Phi^{-1}(u) + \alpha], \quad u \in [0, 1], \]

where \( \alpha \in \mathbb{R} \) and \( \Phi \) is the standard normal distribution with density function:

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \in \mathbb{R}. \]

For \( \alpha > 0 \), \( g_\alpha \) satisfies all the desirable properties of a distortion operator for insurance pricing:

1. \( 0 < g_\alpha(u) < 1 \) for \( u \in [0, 1] \), with \( g_\alpha(0) = \lim_{u \to 0^+} g_\alpha(u) = 0 \) and \( g_\alpha(1) = \lim_{u \to 1^-} g_\alpha(u) = 1 \).
2. $g_\alpha$ is increasing and for $x = \Phi^{-1}(u)$,
\[ g'_\alpha(u) = \frac{\partial g_\alpha(u)}{\partial u} = \frac{\phi(x + \alpha)}{\phi(x)} = e^{-\alpha x - \frac{x^2}{2}} > 0, \quad u \in (0, 1). \]

3. For $\alpha > 0$, $g_\alpha$ is concave and
\[ g''_\alpha(u) = \frac{\partial^2 g_\alpha(u)}{\partial u^2} = -\frac{\alpha \phi(x + \alpha)}{\phi(x)^2} < 0, \quad u \in (0, 1). \]

4. For $\alpha > 0$, $g'_\alpha(u)$ becomes unbounded as $u$ approaches 0.

5. The dual operator $g^*_\alpha(u) = 1 - g_\alpha(1 - u) = g_{-\alpha}(u)$.

6. $g^*_\alpha$ is convex when $g_\alpha$ is concave, as in 3.
For $g_\alpha$ the Choquet integral gives

$$H[X; \alpha] = \int_{-\infty}^{0} \{g_\alpha[S_X(x)] - 1\}dx + \int_{0}^{\infty} g_\alpha[S_X(x)]dx,$$

which enjoys some very nice properties:

1. $H[c; \alpha] = c$ and $H[X + c; \alpha] = H[X; \alpha] + c$, for any constant $c$.

2. $H[bX; \alpha] = bH[X; \alpha]$ while $H[-bX; \alpha] = -bH[X; -\alpha]$, for any $b > 0$. In particular $H[-X; \alpha] = -H[X; -\alpha]$.

3. If $X_1, X_2$ are comonotone $H[X_1 + X_2; \alpha] = H[X_1; \alpha] + H[X_2; \alpha]$. Since two layers of the same risk are comonotone, then

$$H[X_{(a,b)}; \alpha] + H[X_{(b,c)}; \alpha] = H[X_{(a,c)}; \alpha],$$

for any $a < b < c$. 

15
4. For any two variables $X_1$ and $X_2$

$$H[X_1 + X_2; \alpha] \leq H[X_1; \alpha] + H[X_2; \alpha], \quad \text{for } \alpha > 0,$$

$$\geq H[X_1; \alpha] + H[X_2; \alpha], \quad \text{for } \alpha < 0,$$

showing the benefit of diversification.

5. $H[X; \alpha]$ is an increasing function of $\alpha$, with

$$\min[X] \leq H[X; \alpha] \leq \max[X].$$

6. With $\alpha > 0$ for losses and $\alpha < 0$ for assets, $H[X; \alpha]$ preserves 1st and 2nd order stochastic dominance [see Rothschild and Stiglitz (1971)].

7. For $\alpha > 0$, $g'_\alpha(u)$ goes unbounded as $u$ approaches 0. For example, if $X \sim \text{Bernoulli}(\theta)$ then

$$\lim_{\theta \to 0} \frac{H[X; \alpha]}{E[X]} = \lim_{\theta \to 0} \frac{\Phi[\Phi^{-1}(\theta) + \alpha]}{\theta} = g'_\alpha(0) = +\infty.$$
If $X \sim \text{Normal}(\mu, \sigma^2)$ then the ground-up survival function $S_{X\alpha} = g_\alpha \circ S_X$ defines a new $X_\alpha \sim \text{Normal}(\mu + \alpha \sigma, \sigma^2)$, that is

$$H[X; \alpha] = E[X_\alpha] = E[X] + \alpha \sigma[X],$$

which is the usual standard deviation principle.

If $Y \sim \text{Lognormal}(\mu, \sigma^2)$ then the ground-up survival function $S_{Y\alpha} = g_\alpha \circ S_Y$ defines a new $Y_\alpha \sim \text{Lognormal}(\mu + \alpha \sigma, \sigma^2)$.

$g_\alpha$ can be applied to any distribution, but closed forms other than Bernoulli, normal and lognormal do not exist. Numerical evaluations are simple (e.g. in Excel NORMDIST, NORMINV).

The generalization to multivariate variables is possible, simply applying the distortion to the joint distribution function.
3. Applications

3.1 Asset Pricing

Consider asset $i$ with current price $A_i(0)$ and future price $A_i(1)$ after one period.

$$R_i = \frac{A_i(1)}{A_i(0)} - 1$$

is the annual return, assumed normal with mean $E[R_i]$ and standard deviation $\sigma[R_i]$.

Now, if assets can be priced by applying $H[\cdot; -\alpha_i]$ to the present value of the future asset price at some time, then

$$A_i(0) = H\left[\frac{A_i(1)}{1 + r_f}; -\alpha_i\right] = H\left[\frac{A_i(0)(1 + R_i)}{1 + r_f}; -\alpha_i\right],$$

where $r_f$ is the risk-free return compounded annually.
It follows that the risk-adjusted rate of return must coincide with the risk-free rate and hence

\[ H[R_i; -\alpha_i] = E[R_i] - \alpha_i \sigma[R_i] = r_f, \]

which implies [Wang (2000)]:

\[ \alpha_i = \frac{E[R_i] - r_f}{\sigma[R_i]} . \]

A similar implied \( \alpha \) applies to an asset portfolio or, by extension, to the market portfolio \( M \). Under similar assumptions

\[ \alpha_M = \frac{E[R_M] - r_f}{\sigma[R_M]} , \]

which is called the market price of risk (Cummins, 1990) in the expression for the capital market line (CML) in the CAPM.
The idea can be generalized to $T$ periods. For asset $i$ let $R_{it}$ be the annual return in period $t = 1, \ldots, T$ and $A_i(t)$ the price at time $t$. Then

$$R_{it} = \ln \frac{A_i(t)}{A_i(t-1)}.$$ 

Now assume that the $R_{it}$ are iid Normal. The total $T$-period return for asset $i$ is

$$R_i(T) = \ln \frac{A_i(T)}{A_i(0)} = \sum_{t=1}^{T} R_{it},$$

with

$$E[R_i(T)] = \sum_{t=1}^{T} E[R_{it}] = TE[R_i]$$

and

$$V[R_i(T)] = \sum_{t=1}^{T} \{\sigma[R_{it}]\}^2 = T\{\sigma[R_i]\}^2.$$
Again this implies that

\[ H[R_i(T); -\alpha_i] = TE[R_i] - \alpha_i \sqrt{T} \sigma[R_i] = T r_f, \]

and hence [Wang (2000)]:

\[ \alpha_i = \sqrt{T} \left\{ \frac{E[R_i] - r_f}{\sigma[R_i]} \right\}. \]

The idea is easily extended to the continuous time model, with geometric Brownian motion (GBM) prices, \( A_t \) satisfying

\[ \frac{dA_i(t)}{A_i(t)} = \mu_i dt + \sigma_i dW(t). \]
Let $A_i(0)$ be the current asset price. At any future date $T$, the price $A_i(T)$ is solution to the above equation and has a lognormal distribution [Hull (1997)]:

$$\ln \frac{A_i(T)}{A_i(0)} \sim \text{Normal}[(\mu - \frac{\sigma^2}{2})T, \sigma^2T].$$

No-arbitrage with a continuous risk-free return $r_c$ implies:

$$A_i(0) = H[e^{-r_c T} A_i(T); -\alpha_i] = e^{-r_c T} H[A_i(T); -\alpha_i],$$

which implies again [Wang (2000)]:

$$\alpha_i = \frac{(\mu_i - r_c) \sqrt{T}}{\sigma_i}.$$  

Notice that $\alpha_i$ shares many properties with the volatility and is proportional to the square root of the horizon $T$. 

22
3.2 Capital Asset Pricing Model (CAPM)

As seen, the market price of risk $\alpha_M$ in the CML of CAPM links the systematic risk of $X$ to the portfolio risk.

The CAPM measures a portfolio’s risk under the constraint of an expected return level in one compounding period and the assumption of an efficient market.

Mixing a risk-free return $r_f$ with an efficient market portfolio, gives a minimal risk portfolio for a given level of expected return.

Let the expected return and risk of this optimal strategy be $\mu_p$ and $\sigma_p$, respectively. Then the following capital market line (CML) relation holds:

$$\mu_p = r_f + \alpha_M \sigma_p.$$
For asset $i$ with return $R_i \sim \text{Normal}(\mu_i, \sigma_i^2)$ and a market portfolio return $R_M \sim \text{Normal}(\mu_M, \sigma_M)$, consider the security market line (SML):

$$\mu_i = r_f + \beta_i(\mu_M - r_f),$$

where

$$\beta_i = \frac{\text{Cov}[R_i, R_M]}{\sigma_M^2} = \rho_{i,M} \frac{\sigma_i}{\sigma_M}.$$  

Since $\alpha_M = \frac{\mu_M - r_f}{\sigma_M}$, the above SML relation can be rewritten as $\mu_i = r_f + \rho_{i,M} \alpha_M \sigma_i$, which in turn yields:

$$\rho_{i,M} \alpha_M = \frac{\mu_i - r_f}{\sigma_A} = \alpha_i.$$  

Wang (2000) generalizes CAPM to include insurance risks that are not normally distributed, valuing them with $H[.; \alpha]$. 

24
3.3 Black-Scholes Formula

Black-Scholes option pricing formula is often seen as a stop-loss reinsurance contract under a lognormal risk neutral price.

Wang (2000) shows how the distortion operator reproduces Black-Scholes formula under the specific $\alpha$ of the CAPM.

Let $A(t)$ be the current price on day $t$ of an asset and $K$ a European call option strike price, with a right to exercise after $T$ periods.

The payoff of this option can be expressed as

$$A(t + T; K, \infty) = \begin{cases} 0 & \text{if } A(t + T) \leq K \\ A(t + T) - K & \text{if } A(t + T) > K \end{cases}.$$
The expected payoff is then
\[ E[A(t + T; K, \infty)] = \int_0^\infty S_{A(t+T;K,\infty)}(x)dx = \int_K^\infty S_{A(t+T)}(y)dy, \]
while the price under the distortion operator \( g_{-\alpha} \) is
\[ e^{-r_f T}H[A(t + T; K, \infty); -\alpha] = e^{-r_f T} \int_K^\infty [S_{A_{-\alpha}(t+T)}(y)]dy. \]

It easily seen that with a GBM return of rate with mean \( \mu \) and volatility \( \sigma \), the ground-up variables \( A_{-\alpha}(t) \) implies a lognormal distribution
\[ \ln \frac{A'_{t+T}}{A'_t} \sim \text{Normal}[(r_f - \frac{1}{2}\sigma^2)T, \sigma^2T], \quad \text{for} \ \alpha = \frac{(\mu - r_f)\sqrt{T}}{\sigma}. \]

Hence, the distortion and its resulting pricing formula reproduce exactly Black-Scholes’ model.
3.4 Deviance and Tail Index

Both, insurance and financial analysts, use indexes to measure and compare “riskyness”, in the form of a volatility measure, down-side risk, variance, deviance or more generally speaking, a risk measure.

The distortion operator $g_\alpha$ induces a risk loading $H[X; \alpha]$. It is natural to think that it can also be used to define a risk measure.

Since $\min[X] \leq H[X; \alpha] \leq \max[X]$, for any $\alpha \in \mathbb{R}$, and in particular $H[X; 0] = E(X)$ for $\alpha = 0$, Wang (2000) defines right and left deviance measures.
**Definition:** For $\alpha > 0$ fixed and any risk $X$, the right and left deviance indexes are given, respectively, by

$$RD_\alpha[X] = \frac{1}{\alpha} \{H[X; \alpha] - E(X)\}, \quad LD_\alpha[X] = \frac{1}{\alpha} \{E(X) - H[X; -\alpha]\}.$$ 

Note that these deviances measures are function of $\alpha$, but converge to a common limit at $\alpha = 0$:

$$\lim_{\alpha \searrow 0} RD_\alpha[X] = \lim_{-\alpha \nearrow 0} LD_\alpha[X] = \frac{\partial}{\partial \alpha} H[X; \alpha] \bigg|_{\alpha=0},$$

which measures the infinitesimal “force of distortion” at $\alpha = 0$. This motivates our definition of a deviance index.

**Definition:** For $\alpha > 0$ fixed and any risk $X$, the deviance index is given by

$$DI[X] = \frac{\partial}{\partial \alpha} H[X; \alpha] \bigg|_{\alpha=0}.$$ 

Note that by definition of $H[X; \alpha]$, this derivative always exists.
In finance the down-side risk is usually measured in relation to an underlying normal distribution of asset returns.

Wang (2000) uses his right and left deviance indexes to define right and left tail indexes, with the normal distribution serving as a benchmark.

**Definition:** For $\alpha > 0$ fixed and any risk $X$, the right and left tail indexes are given, respectively, by

$$RTI_{\alpha}[X] = \frac{RD_{2\alpha}[X]}{RD_{\alpha}[X]} \quad \text{and} \quad LTI_{\alpha}[X] = \frac{LD_{2\alpha}[X]}{LD_{\alpha}[X]}.$$  

These are meant to be scaleless and differ from his deviance indexes which are dependent on the parameter $\alpha$. 
Example: If $X \sim \text{normal}(\mu, \sigma^2)$ then $H[X; \alpha] = \mu + \alpha \sigma$ and $E(X) = \mu$, for $\alpha, \mu \in \mathbb{R}$ and $\sigma > 0$. It implies that

$$RD_\alpha[X] = \frac{\mu + \alpha \sigma - \mu}{\alpha} = \frac{\mu - \mu + \alpha \sigma}{\alpha} = LD_\alpha[X] = \sigma > 0,$$

reproducing the standard deviation.

Note that the same result is obtained with our deviance index

$$DI[X] = \sigma,$$

a measure always independent of the parameter $\alpha$.

Similarly

$$RTI_\alpha[X] = 1 = LTI_\alpha[X],$$

showing how normal tails are used here as a benchmark; index values larger than 1 indicate fatter than normal tails.
Example: If $X \sim \lognormal(\mu, \sigma^2)$ then $H[X; \alpha] = e^{(\mu + \alpha \sigma) + \frac{\sigma^2}{2}}$ and $E(X) = e^{\mu + \frac{\sigma^2}{2}}$, for $\alpha, \mu \in \mathbb{R}$ and $\sigma > 0$. Hence

$$RD_{\alpha}[X] = \frac{e^{(\mu + \alpha \sigma) + \frac{\sigma^2}{2}} - e^{\mu + \frac{\sigma^2}{2}}}{\alpha} = e^{\mu + \frac{\sigma^2}{2}} \frac{(e^{\alpha \sigma} - 1)}{\alpha},$$

while the left deviance index has no physical interpretation here:

$$LD_{\alpha}[X] = e^{\mu + \frac{\sigma^2}{2}} \frac{(1 - e^{-\alpha \sigma})}{\alpha}.$$

Note that these deviances depend on the parameter $\alpha$ and differ from the standard deviation:

$$\sigma[X] = e^{\mu + \frac{\sigma^2}{2}} \sqrt{(e^{\sigma^2} - 1)}.$$

By contrast, our deviance index is closer in value to $\sigma[X]$

$$DI[X] = \sigma e^{\mu + \frac{\sigma^2}{2}}.$$
Finally, Wang’s tail indexes are given here by:

\[ RTI_\alpha[X] = \frac{e^{\alpha \sigma} + 1}{2} \quad \text{and} \quad LTI_\alpha[X] = \frac{1 + e^{-\alpha \sigma}}{2}. \]

Oddly enough, here \( \sigma \) values need to be such that \( e^{\alpha \sigma} > 1 \) for lognormal tails to be fatter than normal.

This illustrates the undesirable dependence of Wang’s tail index on the parameter \( \alpha \).

Alternative tail indexes, based on higher derivatives of \( H[X; \alpha] \) at \( \alpha = 0 \), are being studied.
Example: For $X \sim \text{Bernoulli}(\theta_0)$, $H[X; \alpha] = \theta_\alpha = \Phi[\Phi^{-1}(\theta_0) + \alpha]$ and $E[X] = \theta_0$, where $\alpha \in \mathbb{R}$ and $0 \leq \theta_0 \leq 1$. Hence

$$RD_\alpha[X] = \frac{\Phi[\Phi^{-1}(\theta_0) + \alpha] - \theta_0}{\alpha} = \frac{\theta_\alpha - \theta_0}{\alpha},$$

while again, the left deviance index has no physical interpretation:

$$LD_\alpha[X] = \frac{\theta_0 - \Phi[\Phi^{-1}(\theta_0) - \alpha]}{\alpha} = \frac{\theta_0 - \theta_{-\alpha}}{\alpha}.$$

These deviance measures also differ from the standard deviation $\sigma[X] = \sqrt{\theta_0(1-\theta_0)}$ and depend on the parameter $\alpha$.

By contrast, our deviance index is given by:

$$DI[X] = \phi[\Phi^{-1}(\theta_0)] = \frac{1}{\sqrt{2\pi}}e^{-\frac{[\Phi^{-1}(\theta_0)]^2}{2}}.$$
Finally, Wang’s tail indexes

\[ RTI_\alpha[X] = \frac{\Phi[\Phi^{-1}(\theta_0) + 2\alpha] - \theta_0}{2\{\Phi[\Phi^{-1}(\theta_0) + \alpha] - \theta_0\}} = \frac{\theta_{2\alpha} - \theta_0}{2(\theta_\alpha - \theta_0)} \]

and

\[ LTI_\alpha[X] = \frac{\theta_0 - \Phi[\Phi^{-1}(\theta_0) - 2\alpha]}{2\{\theta_0 - \Phi[\Phi^{-1}(\theta_0) - \alpha]\}} = \frac{\theta_0 - \theta_{-2\alpha}}{2(\theta_0 - \theta_{-\alpha})}, \]

depend on \( \alpha \) in an intricate way.
4. Dynamic Coherent Measures

Wang’s use of distortion operators and Choquet integrals has been embedded in the framework of coherent risk measures [Artzner, Delbaen, Eber and Heath (1999)].

In Balbás, Garrido and Mayoral (2002) we assume a dynamic framework which reflects the stochastic behavior of (insurance) risks and (financial) pay-offs.

As a special application, we study the extensions of VaR (Value at Risk) and TCE (Tail Conditional Expectation) to this dynamic setting.
Bibliography:


