Modeling and Monitoring Risk Acceptability in Markets: 
The Case of the Credit Default Swap Market

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Outline of Presentation

- Discussion of Fundamental Differences between Economics and Financial Economics.

- Cones of Acceptable Risks and Two Price Economies.
- Implications of No Arbitrage in Two Price Economies.
- Valuation by Nonlinearly Discounted Nonlinear Martingales.
- Modeling Rate Measure Change Interactions.
- CDS Pricing to Market Acceptability.

Risk Acceptability through the Crisis as seen by the CDS Market.
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There is always risk exposure to loss and the financial system by defining the level of acceptable risk exposure, determines the size of the real economy.
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- They are the positive alpha trades earning above and beyond risk compensation.
- They are defined by the half space of random variables making an angle below $90^0$ with the risk neutral density.
- As a half space it is an example of the largest possible convex set containing the nonnegative random variables.
For a Financial economy we follow Artzner, Delbaen, Eber and Heath (1999) to define acceptable risks as a proper convex cone containing the nonnegative random variables that is in general much smaller than a half space.
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Such cones are defined by a family of probability measures $Q \in \mathcal{M}$ called test measures or scenarios and $X$ is acceptable or $X \in \mathcal{A}$ just if

$$E^Q[X] \geq 0, \text{ all } Q \in \mathcal{M}.$$
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We have a two price economy with nonlinear pricing operators that are concave and convex for bid and ask respectively.
Consider on the other hand a prevailing two price economy that does not permit arbitrage.
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Traded are cash flows \((C(u), u > 0)\) for time \(t\) bid and ask prices \(b_C(t), a_C(t)\) with associates zero cost cash flows

\[
Z_b(u) = 0, u < t; \quad Z_b(t) = b_C(t) - C(t); \quad Z_b(u) = -C(u), u > t
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Z_a(u) = 0, u < t; \quad Z_a(t) = -a_C(t) + C(t)b; \quad Z_b(u) = C(u), u > t
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By no arbitrage there exists a positive process \(\theta(t)\) such that for cash flows \(X(t)\) bounded above by a zero cost cash flow we have

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E \left[ \sum_{t} \theta(t)X(t) \right] \leq 0.
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Let \(\mathcal{M}\) be the set of all such positive separating processes.
It follows that the best bid and ask prices are given by

\[ b_C(t) = \inf_{\theta \in M} E_t \left[ \sum_{u > t} \frac{\theta(u)}{\theta(t)} C(u) \right] \]

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In particular we observe that

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Invoking dynamic consistency we write

\[ b_C(t) = \inf_{\theta \in \mathcal{M}} E_t \left[ C(t) + \frac{\theta(t+1)}{\theta(t)} b_C(t+1) \right] \]

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In particular we observe that

\[ a_C(t) = -b_{-C}(t). \]
Define $\Gamma(t)$ adapted to $t - 1$ by

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\begin{align*}
\Gamma(0) &= 1 \\
\Gamma(t) &= \Gamma(t - 1) \frac{E_{t-1}[\theta(t)]}{\theta(t - 1)}
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$$M(t) = \frac{\theta(t)}{\Gamma(t)}$$
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Observe that $M(t) = \frac{\theta(t)}{\Gamma(t)}$ is a martingale and we may decompose

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\theta(t) = \Gamma(t) M(t).
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Hence

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It follows that $\Gamma(t)$ is decreasing and separating hyperplanes are products of discount function $\Gamma(t)$ and martingales $M(t)$. 
We may then write on incorporating densities into measure changes that

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- Otherwise each product has its own and possibly unique trade direction specific discount function and measure change.
- In the financial industry trading desks already now use product specific discount functions and pricing models.
- We have multicurve n-way cooking for the construction of discount curves.
- Hence the industry is already there, it just taking us time to catch up and see the underlying logic.
Suppose first we may put discounting issues aside and we only have to model measure changes in defining acceptability.
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The first is that we ask for additivity for comonotone risks whereby if $X, Y$ have no negative comovements then we require

$$b(X + Y) = b(X) + b(Y).$$
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Technically this condition requires the set of test measures to be the core of a convex game.
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Under these two conditions there exists a concave distribution function $\Psi$ from the unit interval to itself such that for a random variable $X$ with distribution function $F(x)$

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The change of measure $\Psi'(F(x))$ reweights losses upwards and discounts gains.
An explicit concave distortion termed minmaxvar

- Cherny and Madan (2009) introduced the minmaxvar distortion defined by

\[ \Psi^\gamma(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}. \]
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Under this distortion a Gaussian random variable with a mean return of 6.25% and a volatility of 15% is acceptable at the stress level of $\gamma = 0.25$. 
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At an infinite stress level only arbitrages are acceptable.
Define the stress conditional distorted expectation by

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We are accustomed in finance when working with cash flows under the physical measure to discount at the risk rate that exceeds the risk free rate by the risk compensation.
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Once we change measure to a high stress level we could approach a risk neutral measure and then we discount at the risk free rate.
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The interaction is modelled by letting the discount rate depend on the stress level.
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for a distribution function \( H \) on the unit interval, and a maximum stress level \( \bar{\gamma} \).
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for a distribution function \( H \) on the unit interval, and a maximum stress level \( \gamma \).

Given a parameter for the speed at which we reduce the rate in response to the stress level this is a four parameter model for the cone of acceptability.
The bid price is obtained by requiring all the stress specific discounted expectations to be positive for $0 \leq \gamma \leq \gamma$ and

$$b(X) = \inf_{0 \leq \gamma \leq \gamma} \exp(-r(\gamma)T)h_X(\gamma).$$
The bid price is obtained by requiring all the stress specific discounted expectations to be positive for $0 \leq \gamma \leq \overline{\gamma}$ and
\[ b(X) = \inf_{0 \leq \gamma \leq \overline{\gamma}} \exp(-r(\gamma)T) h_X(\gamma). \]

It is observed in Madan (2014) that for a loan
\[ a(X) = \exp(-r(\overline{\gamma})T) h_X(\overline{\gamma}). \]
For a CDS with default probability $p$ we evaluate for recovery rate $R$ that

$$h_X(\gamma) = 1 - (1 - R)\Psi^\gamma(p).$$
CDS Pricing to Market Acceptability

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We may then explicitly compute the bid and ask prices and take as representative for the risk neutral price the mid quote $m$. 

$$e^{\lambda} = \frac{1}{T \ln m \exp(r_{\min}T) R} \psi^\gamma(p).$$

$$c = e^{\lambda} (1 - R) = \frac{1}{R} \frac{1}{T \ln m \exp(r_{\min}T) R}.$$
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- For a risk neutral hazard rate $\tilde{\gamma}$ we write

$$m = \exp(-r_{\min}T)(R \left(1 - e^{-\tilde{\lambda}T}\right) + e^{-\tilde{\lambda}T})$$
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- For a risk neutral hazard rate $\tilde{\gamma}$ we write
  \[ m = \exp(-r_{\min}T)(R(1 - e^{-\tilde{\lambda}T}) + e^{-\tilde{\lambda}T}) \]
- or
  \[ \tilde{\lambda} = -\frac{1}{T} \ln \left( \frac{m \exp(r_{\min}T) - R}{1 - R} \right) \]
For a CDS with default probability $p$ we evaluate for recovery rate $R$ that

$$h_X(\gamma) = 1 - (1 - R)\Psi^\gamma(p).$$

We may then explicitly compute the bid and ask prices and take as representative for the risk neutral price the mid quote $m$.

For a risk neutral hazard rate $\tilde{\gamma}$ we write

$$m = \exp(-r_{\text{min}} T)(R \left(1 - e^{-\tilde{\lambda}T}\right) + e^{-\tilde{\lambda}T})$$

or

$$\tilde{\lambda} = -\frac{1}{T} \ln \left(\frac{m \exp(r_{\text{min}} T) - R}{1 - R}\right)$$

hence

$$c = \tilde{\lambda}(1 - R) = -\frac{1 - R}{T} \ln \left(\frac{m \exp(r_{\text{min}} T) - R}{1 - R}\right).$$
We thus have

$$cds\ quote = C(p, r_{\text{min}}, r_{\text{max}}, \bar{\gamma}, A).$$
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$$cds \text{ quote} = C(p, r_{\text{min}}, r_{\text{max}}, \bar{\gamma}, A).$$

This is a model pricing the CDS to market acceptability using a conic NLDNLM definition of acceptability.
General properties of the model

- We take a range of values for the five inputs, compute the cds quote with a five percent multiplicative error and then regress the cds quote on the inputs and the cross products.
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We observe that the credit spread is increasing concave in $p$ and decreasing convex in $R$. Under risk neutral pricing the credit is linear in $R$ and increasing convex like $\ln(1+p)$. These are testable differences.
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- These are testable differences.
Data Description

- We first tested parameter identification by generating simulated data from the model with a five percent error volatility and observing that maximum likelihood estimation was able to recover the parameter values.

We obtained from the Risk Management Institute of NUS daily data on default probabilities over the period January 23, 2006 to January 31, 2014 for six US Banks, JPM, BAC, WFC, C, MS and GS. We also obtained data on daily CDS quotes at market close for these banks for the same period. The data set had 2040 observations for PD's and CDS rates for the six banks. We present in Tables 2 and 3 respectively the summary statistics for these data sets. The summary statistics are the mean, the standard deviation (std), skewness (skw), kurtosis (krt), the maximum (max), the minimum (min) and the quantiles at 1, 10, 25, 50, 75, 90 and 99 percent.
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- The summary statistics are the mean, the standard deviation (std), skewness (skw), kurtosis (krt), the maximum (max), the minimum (min) and the quantiles at 1, 10, 25, 50, 75, 90 and 99 percent.
Table 2
Summary Statistics for PD’s

<table>
<thead>
<tr>
<th></th>
<th>JPM</th>
<th>BAC</th>
<th>WFC</th>
<th>C</th>
<th>MS</th>
<th>GS</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0416</td>
<td>0.0671</td>
<td>0.0367</td>
<td>0.0750</td>
<td>0.0822</td>
<td>0.0526</td>
</tr>
<tr>
<td>std</td>
<td>0.0168</td>
<td>0.0427</td>
<td>0.0316</td>
<td>0.0841</td>
<td>0.0416</td>
<td>0.0334</td>
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<tr>
<td>skw</td>
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<td>1.3489</td>
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<tr>
<td>krt</td>
<td>3.4235</td>
<td>13.903</td>
<td>5.0015</td>
<td>25.753</td>
<td>3.9246</td>
<td>3.7467</td>
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<tr>
<td>max</td>
<td>0.1066</td>
<td>0.3905</td>
<td>0.1622</td>
<td>0.8246</td>
<td>0.3119</td>
<td>0.1501</td>
</tr>
<tr>
<td>min</td>
<td>0.0137</td>
<td>0.0156</td>
<td>0.0084</td>
<td>0.0178</td>
<td>0.0298</td>
<td>0.0170</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0147</td>
<td>0.0196</td>
<td>0.0090</td>
<td>0.0214</td>
<td>0.0320</td>
<td>0.0186</td>
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<td>0.1</td>
<td>0.0204</td>
<td>0.0247</td>
<td>0.0101</td>
<td>0.0254</td>
<td>0.0406</td>
<td>0.0220</td>
</tr>
<tr>
<td>0.25</td>
<td>0.0300</td>
<td>0.0355</td>
<td>0.0120</td>
<td>0.0317</td>
<td>0.0514</td>
<td>0.0292</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0391</td>
<td>0.0595</td>
<td>0.0277</td>
<td>0.0482</td>
<td>0.0677</td>
<td>0.0409</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0497</td>
<td>0.0900</td>
<td>0.0437</td>
<td>0.0825</td>
<td>0.1033</td>
<td>0.0638</td>
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<tr>
<td>0.9</td>
<td>0.0679</td>
<td>0.1111</td>
<td>0.0865</td>
<td>0.1281</td>
<td>0.1501</td>
<td>0.1204</td>
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<tr>
<td>0.99</td>
<td>0.0877</td>
<td>0.2390</td>
<td>0.1370</td>
<td>0.3944</td>
<td>0.1922</td>
<td>0.1426</td>
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</tbody>
</table>
## Table 3
Summary Statistics for CDS Rates

<table>
<thead>
<tr>
<th></th>
<th>JPM</th>
<th>BAC</th>
<th>WFC</th>
<th>C</th>
<th>MS</th>
<th>GS</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0082</td>
<td>0.0135</td>
<td>0.0083</td>
<td>0.0152</td>
<td>0.0182</td>
<td>0.0143</td>
</tr>
<tr>
<td>std</td>
<td>0.0042</td>
<td>0.0097</td>
<td>0.0050</td>
<td>0.0115</td>
<td>0.0143</td>
<td>0.0093</td>
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<tr>
<td>skew</td>
<td>0.1924</td>
<td>0.9008</td>
<td>0.7180</td>
<td>1.2940</td>
<td>2.1173</td>
<td>0.8278</td>
</tr>
<tr>
<td>kurt</td>
<td>2.6691</td>
<td>3.7609</td>
<td>4.6826</td>
<td>5.7935</td>
<td>14.242</td>
<td>3.4000</td>
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<tr>
<td>max</td>
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<td>0.0311</td>
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<td>0.1621</td>
<td>0.0588</td>
</tr>
<tr>
<td>min</td>
<td>0.0014</td>
<td>0.0009</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0015</td>
<td>0.0022</td>
</tr>
<tr>
<td>.01</td>
<td>0.0014</td>
<td>0.0010</td>
<td>0.0009</td>
<td>0.0010</td>
<td>0.0020</td>
<td>0.0024</td>
</tr>
<tr>
<td>.1</td>
<td>0.0020</td>
<td>0.0014</td>
<td>0.0012</td>
<td>0.0013</td>
<td>0.0024</td>
<td>0.0027</td>
</tr>
<tr>
<td>.25</td>
<td>0.0056</td>
<td>0.0067</td>
<td>0.0048</td>
<td>0.0075</td>
<td>0.0098</td>
<td>0.0087</td>
</tr>
<tr>
<td>.5</td>
<td>0.0084</td>
<td>0.0127</td>
<td>0.0085</td>
<td>0.0142</td>
<td>0.0151</td>
<td>0.0129</td>
</tr>
<tr>
<td>.75</td>
<td>0.0108</td>
<td>0.0173</td>
<td>0.0109</td>
<td>0.0207</td>
<td>0.0240</td>
<td>0.0182</td>
</tr>
<tr>
<td>.9</td>
<td>0.0140</td>
<td>0.0276</td>
<td>0.0142</td>
<td>0.0274</td>
<td>0.0385</td>
<td>0.0288</td>
</tr>
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<td>0.0248</td>
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<td>0.0543</td>
<td>0.0400</td>
</tr>
</tbody>
</table>
We estimated the model using CDS and PD data for 21 days prior to and including the estimation date that was taken at 10 day intervals starting at February 20, 2006 and finishing at January 31, 2014 for a total of 203 estimations.
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There are four parameters for the cone of acceptability, the maximal and minimal rate employed in time value discounting, the maximal stress level being demanded by the market, and the speed at which one moves down from the maximal rate to the minimal rate as the stress level is raised.
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Figure 1 presents the maximal discount rates estimated over the time period along with a smoothed version of the same.
Figure: Maximal discount rates employed in evaluating contract acceptability.
Maximal Stress Level

Figure: Maximal Stress Levels Reflected in CDS rates for the Financial Sector.

Dilip B. Madan (Robert H. Smith School of Business)

Risk Acceptability in Markets

May 29 2014 RiskLab, Madrid

25 / 27
Risk acceptability in markets is modeled as a minimal discounted distorted expectation taken across a range of stress level for the distortion.
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Interactions between discounting and stress are accommodated via a reduction of discount rates for the higher stress levels.
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The parameters for acceptability represent a maximal and minimal discount rate for time value accounting, a maximal stress level being employed and the speed of rate reduction in response to an increase in the stress level used.
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Interactions between discounting and stress are accommodated via a reduction of discount rates for the higher stress levels.

The parameters for acceptability represent a maximal and minimal discount rate for time value accounting, a maximal stress level being employed and the speed of rate reduction in response to an increase in the stress level used.

The model is explicitly employed to relate physical default probabilities to credit default swap prices quoted in markets.
Data on default probabilities for the six major US banks over the period January 2006 to January 2014 obtained from the Risk Management Institute of the National University of Singapore are employed to estimate from market quotes on CDS rates, the cone parameters and market implied recovery rates through the period under investigation.
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We observe that the financial crisis saw an increase in the maximal discount rate and its spread over the minimal rate along with an increase in the maximal stress level being demanded for acceptability and a stable pattern for the speed of rate adjustment through the period.
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The maximal rate, rate spread and stress levels have come down but with periods in the interim where they have peaked as they did in the crisis.
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The maximal rate, rate spread and stress levels have come down but with periods in the interim where they have peaked as they did in the crisis.

Recovery rates have been oscillating and they have come down but have recovered somewhat towards the end.