Modeling and Monitoring Risk Acceptability in Markets: The Case of the Credit Default Swap Market

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Outline of Presentation

- Discussion of Fundamental Differences between Economics and Financial Economics.

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- Risk Acceptability through the Crisis as seen by the CDS Market.

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- In financial economies with forward state contingent trading such clearing cannot be guaranteed.
- There is always risk exposure to loss and the financial system by defining the level of acceptable risk exposure, determines the size of the real economy.

Acceptable Risks in Economics

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- As a half space it is an example of the largest possible convex set containing the nonnegative random variables.

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- We have a two price economy with nonlinear pricing operators that are concave and convex for bid and ask respectively.

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- Traded are cash flows $(C(u), u > 0)$ for time t bid and ask prices $b_C(t)$, $a_C(t)$ with associates zero cost cash flows

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\begin{aligned}
Z_b(u) &= 0, u < t;\ Z_b(t) = b_C(t) - C(t);\ Z_b(u) = -C(u), u > t \\
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- By no arbitrage there exists a positive process $\theta(t)$ such that for cash flows $X(t)$ bounded above by a zero cost cash flow we have

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- Let $\mathcal{M}$ be the set of all such positive separating processes.

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- It follows that the best bid and ask prices are given by

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b_C(t) &= \inf_{\theta \in \mathcal{M}} E_t\left[\sum_{u>t} \frac{\theta(u)}{\theta(t)} C(u)\right] \\
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- In particular we observe that

$$a_C(t) = -b_{-C}(t).$$

Towards Nonlinearly Discounted Nonlinear Martingales I

- Define $\Gamma(t)$ adapted to $t-1$ by

$$\begin{aligned}\Gamma(0) &= 1 \\ \Gamma(t) &= \Gamma(t-1)\frac{E_{t-1}[\theta(t)]}{\theta(t-1)}\end{aligned}$$

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- is a martingale and we may decompose

$$\theta(t) = \Gamma(t)M(t).$$

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- Consider buying a pure discount bond at $t-1$, maturing at t for the ask price α and cash flow

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- It follows that $\Gamma(t)$ is decreasing and separating hyperplanes are products of discount function $\Gamma(t)$ and martingales $M(t)$.

Valuation by NLDNLM

- We may then write on incorporating densities into measure changes that

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- Hence the industry is already there, it just taking us time to catch up and see the underlying logic.

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- Technically this condition requires the set of test measures to be the core of a convex game.

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- The change of measure $\Psi'(F(x))$ reweights losses upwards and discounts gains.

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- At an infinite stress level only arbitrages are acceptable.

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- The interaction is modelled by letting the discount rate depend on the stress level.

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- for a distribution function H on the unit interval, and a maximum stress level $\overline{\gamma}$.
- Given a parameter for the speed at which we reduce the rate in response to the stress level this is a four parameter model for the cone of acceptability.

Bid Price and Acceptability

- The bid price is obtained by requiring all the stress specific discounted expectations to be positive for $0 \leq \gamma \leq \overline{\gamma}$ and

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- It is observed in Madan (2014) that for a loan

$$a(X) = \exp\left(-r(\overline{\gamma})T\right) h_X(\overline{\gamma}).$$

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- This is a model pricing the CDS to market acceptability using a conic NLDNLM definition of acceptability.

General properties of the model

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- The summary statistics are the mean, the standard deviation (std), skewness (skw), kurtosis (krt), the maximum (max), the minimum (min) and the quantiles at 1, 10, 25, 50 75, 90 and 99 percent.

PD summary

Table 2

Summary Statistics for PD's

	JPM	BAC	WFC	C	MS	GS
mean	0.0416	0.0671	0.0367	0.0750	0.0822	0.0526
std	0.0168	0.0427	0.0316	0.0841	0.0416	0.0334
skw	0.7619	2.4142	1.6174	4.0518	1.1710	1.3489
krt	3.4235	13.903	5.0015	25.753	3.9246	3.7467
max	0.1066	0.3905	0.1622	0.8246	0.3119	0.1501
min	0.0137	0.0156	0.0084	0.0178	0.0298	0.0170
0.01	0.0147	0.0196	0.0090	0.0214	0.0320	0.0186
0.1	0.0204	0.0247	0.0101	0.0254	0.0406	0.0220
0.25	0.0300	0.0355	0.0120	0.0317	0.0514	0.0292
0.5	0.0391	0.0595	0.0277	0.0482	0.0677	0.0409
0.75	0.0497	0.0900	0.0437	0.0825	0.1033	0.0638
0.9	0.0679	0.1111	0.0865	0.1281	0.1501	0.1204
0.99	0.0877	0.2390	0.1370	0.3944	0.1922	0.1426

CD Summary

Table 3

Summary Statistics for CDS Rates

	JPM	BAC	WFC	C	MS	GS
mean	0.0082	0.0135	0.0083	0.0152	0.0182	0.0143
std	0.0042	0.0097	0.0050	0.0115	0.0143	0.0093
skw	0.1924	0.9008	0.7180	1.2940	2.1173	0.8278
krt	2.6691	3.7609	4.6826	5.7935	14.242	3.4000
max	0.0233	0.0493	0.0311	0.0651	0.1621	0.0588
min	0.0014	0.0009	0.0008	0.0009	0.0015	0.0022
.01	0.0014	0.0010	0.0009	0.0010	0.0020	0.0024
.1	0.0020	0.0014	0.0012	0.0013	0.0024	0.0027
.25	0.0056	0.0067	0.0048	0.0075	0.0098	0.0087
.5	0.0084	0.0127	0.0085	0.0142	0.0151	0.0129
.75	0.0108	0.0173	0.0109	0.0207	0.0240	0.0182
.9	0.0140	0.0276	0.0142	0.0274	0.0385	0.0288
.99	0.0182	0.0411	0.0248	0.0587	0.0543	0.0400

Estimation Design

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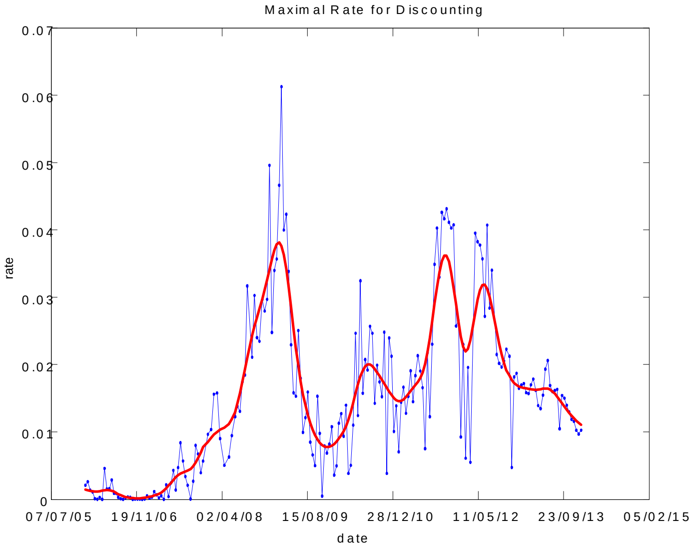
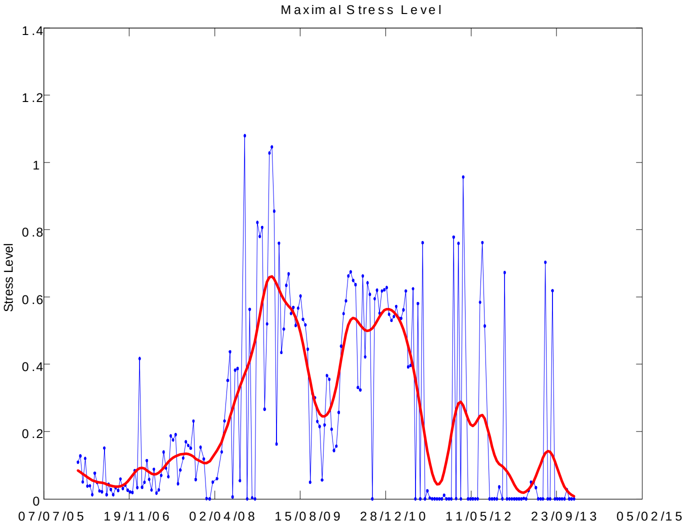
- Figure 1 presents the maximal discount rates estimated over the time period along with a smoothed version of the same.

Maximal Discount Rate



Maximal Stress Level



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- The model is explicitly employed to relate physical default probabilities to credit default swap prices quoted in markets.

Conclusion II

- Data on default probabilities for the six major US banks over the period January 2006 to January 2014 obtained from the Risk Management Institute of the National University of Singapore are employed to estimate from market quotes on CDS rates, the cone parameters and market implied recovery rates through the period under investigation.

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- The maximal rate, rate spread and stress levels have come down but with periods in the interim where they have peaked as they did in the crisis.
- Recovery rates have been oscillating and they have come down but have recovered somewhat towards the end.