New derivatives based structures

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The risk taker's viewpoint

- When investing in a portfolio A of tradable assets (generally involving also derivatives and combinations thereof) a rational investor is in fact implicitly making one of two assumptions on probability.

1. If C(t) denotes the value of cash account accruing at the risk free rate,

   $$\text{Probability}\ \{\ A(t)/C(t) \geq A(0)\ \} = 1 \quad \text{and} \quad \text{Probability}\ \{\ A(t)/C(t) > A(0)\ \} > 0,$$

   in the risk neutral probability. I.e., at at least a time t in the future, A will certainly not under-perform the cash account and in fact, with non-zero probability, will outperform it.

   In this case A provides a risk-free profit; the arbitrage opportunity arises from mis-pricing of A(0) within the risk neutral measure.

2. In the "real" measure $\quad \text{Expectation}[\ A(t)/C(t)] > A(0);$

   or in fact, and preferably, finer assumptions on the shape of the "real" probability distribution of A(t) as compared to the risk neutral distribution. (For example, the assumption that the real distribution is definitely shifted and sharply skewed to the right of the risk neutral one, thus making positive outcomes more probable).

The risk taker's viewpoint

- An investor's decision process goes roughly follows:

  1. Starting from a view on a set of market prices or parameters, identification of the tradable assets – often derivatives – that allow to capture the expected excess value.

  2. Formulation of a practicable trading strategy that responds to rigid risk management criteria.

  3. Package the strategy that most directly exhibits performance under back-testing and stress scenarios.

- Most standard structured derivatives contracts are defined by a sequence of pre-defined payoffs, and valued via risk neutral expectation. Risk measures are typically not constant. It is not obvious how a product could be configured so that, for example, the effect of a future gapping event (e.g., a jump in relevant market variables), or the standard deviation of the contract over a future time window, could be a priori quantified.

- Derivatives desks at investment banks have thus been increasingly moving into the space naturally occupied by funds (asset managers, hedge fund managers, etc). This should not be entirely surprising, as these desks are in fact naturally equipped with the analytic tools for the strategy formulation, and they can take upon themselves and manage the associated execution risks.

Strategy Indices: Purpose

- A **strategy index** is a tradable asset constructed not in terms of a sequence of statically defined payoffs, but rather defined by a sequence of trading operations based on predetermined quantitative criteria. One may think of it as a virtual fund whose decisions and actions are based on algorithms.
- It can be naturally configured so to maintain a predetermined risk exposure.
- **Alpha versus beta character:** to pay tribute to conventional jargon. Alpha is the return that can be generated by managing market positions while preserving both neutrality and low correlation with respect to a set of "main" market indicators (such as: level of interest rates in major currencies; major equity, commodity and bond indices, etc.) to which an investor is likely to already be "beta"-exposed. "Portable" means that the alpha- or beta- generating strategy can be entered independently of any other pre-existing positions; it can hence be embedded into an unrelated structure or overlaid to a portfolio. A portable strategy is most likely achieved through a synthetic formulation, such as an index.
- The value of a strategy index is by construction path-dependent: it depends on how markets evolve between start to end date.

Strategy Indices: Construction

- **Investment Portfolio IP:** at any time t, a linear combinations of (units of) tradable assets A_i of the form

$$IP(t) = \Sigma_i \alpha_i(t) A_i(t)$$

We assume here for simplicity that all assets are entered with no upfront payments (e.g.,swaps).

- **Cash account C:** at any time t, an amount of cash C(t). The account is initialized with a cash injection, say

$$C(0) = 100,$$

it accrues at the risk-free rate r and is recursively defined so to collect all gains and losses realised in the investment portfolio. It serves as a collateral for the risky investment portfolio position.

- **Rolling schedule:** a list of times

$$T_0 \equiv 0 < T_1 < T_2 < \dots \qquad \text{with} \qquad T_i - T_{i-1} = \Delta T$$

at which the position on the investment portfolio is rolled over.

Strategy Indices: Construction

- **Leverage λ:** a function λ(t) expressing the proportion between amount put a risk in the investment portfolio and cash collateral. Generally this is a function of one or more relevant market variables at t:

$$\lambda(t) = \lambda(t;\, \text{market}(t))$$

- **Rolling mechanism and the index recursion:** On every rebalancing date T_i the investment portfolio position is closed with a cash gain or loss, and a new investment portfolio position is entered in a notional amount

$$\lambda(T_i)\, \max[I(T_i),0]$$

The **strategy index** value is hence given by

$$I(t) = \underbrace{\boxed{I(T_{i-1})\,[1+r(t-T_{i-1})]}}_{C(t)} + \lambda(T_{i-1})\, \max[\,I(T_{i-1}),0]\; IP(t) \qquad \text{for} \qquad T_{i-1} < t \le T_i$$

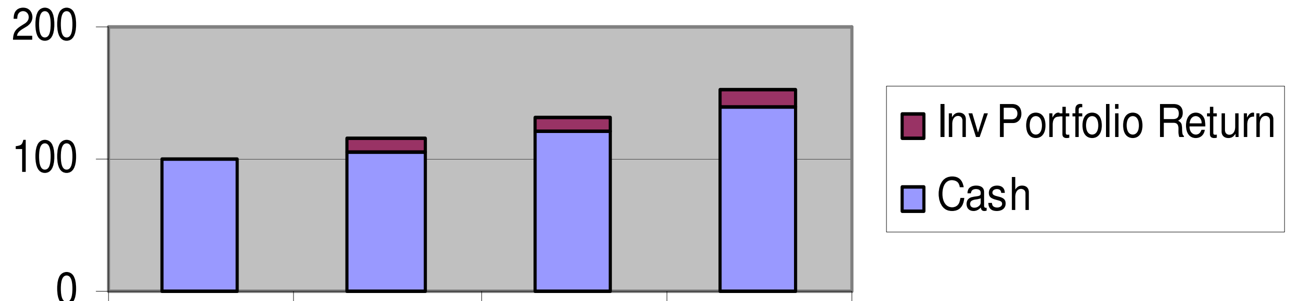
If V_i is the unwinding cash value of the investment portfolio at T_i, the values on rolling dates satisfy the recursion

$$I(T_i) = I(T_{i-1})\,(1+r\Delta T) + \lambda(T_{i-1})\, \max[I(T_{i-1}),0]\; V_i \qquad \text{with} \quad I(0)=C(0)=100.$$

Strategy Indices: Construction

- Example: a strategy index with constant leverage function and yearly spaced rolling dates

Progression of Index value with yearly rebalancing (5% risk-free rate and 10% Inv Portfolio growth rate)



	0	1	2	3
Inv Portfolio Return	0	10	11.5	13.225
Cash	100	105	120.75	138.8625

time in years

Strategy Indices: The leverage function

The leverage function λ is chosen according to risk/return criteria. Here are a few practical possibilities:

1. **Controlling the gap risk:** the index crashes to near zero or below at time t in the interval $(T_{i-1},T_i]$ whenever a loss of size $|\Delta IP|$ occurs so that

$$\lambda(T_{i-1})\ |\Delta IP| \geq 1.$$

   Fixing a gap "cushion" $|\Delta IP|$ bounds the leverage by the condition $\lambda \leq |\Delta IP|$.

2. **Bounding the volatility:** The IP(t) price process calibrated on data at T_{i-1}, together with the risk-free rate process and their correlation, determines the standard deviation of $I(T_i)$ as a function of $\lambda(T_{i-1})$. The latter can be chosen so that the standard deviation should be lower than a given bound.

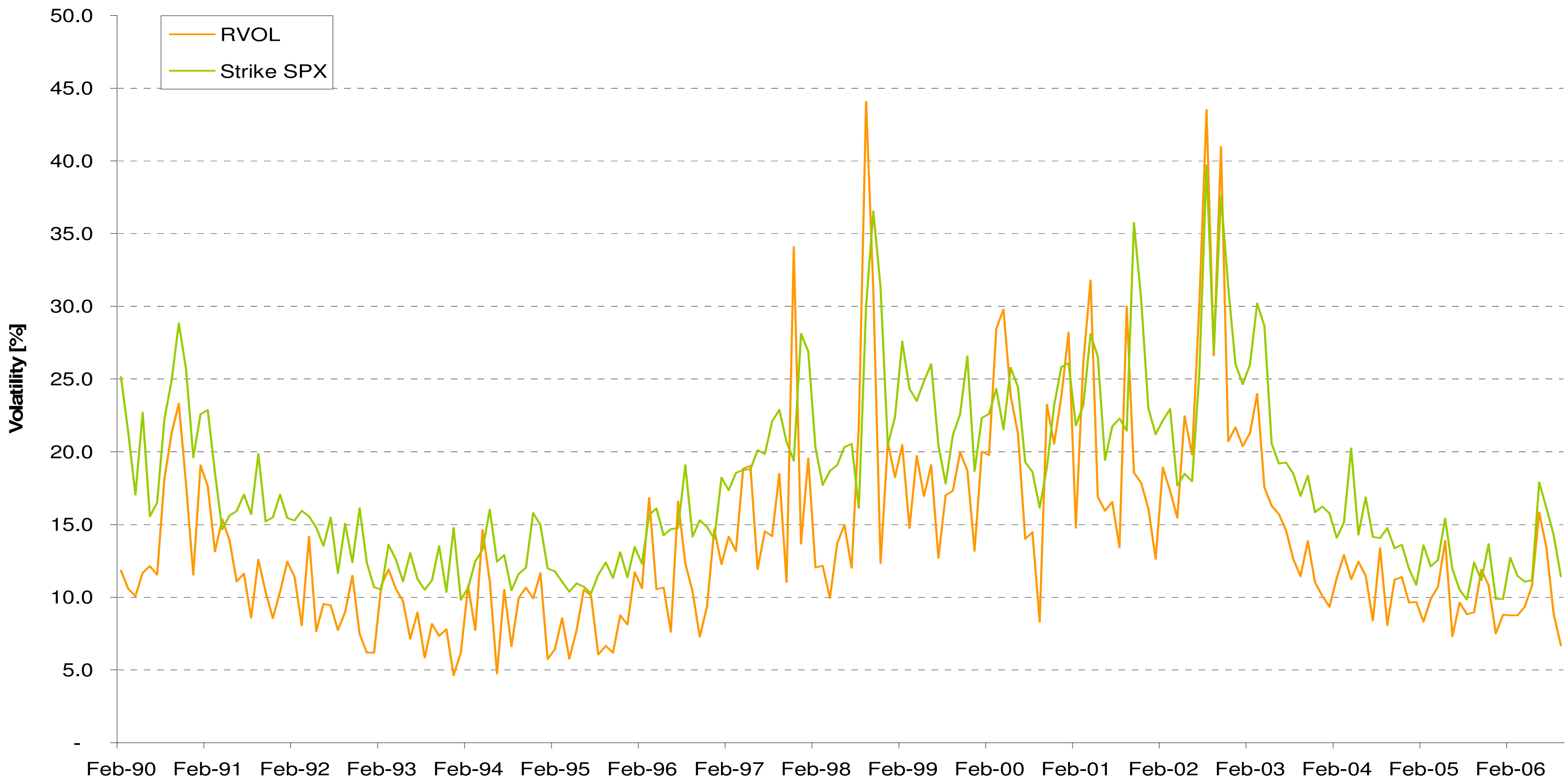
3. **Opportunity weighting:** A criterion that could be superimposed to the above. The amount λ to be put at risk at T_{i-1} can be constructed as a function of the then current (and/or past) values of relevant market indicators. A simple example would be that of a step function that moves from 1 to 0 whenever the value of a certain market indicator at T_{i-1} falls below a given barrier.

Search for opportunities: Equity Volatility arbitrage

- Is there a stable difference between market assets' volatilities and the corresponding realized volatilities. In other words, are options mis-priced, thus introducing a systemic discrepancy between the "arbitrage-free" risk neutral probability and the real world probability?
- Not for all assets.
- Equity: Typically market makers/traders are net sellers of short term volatility/options given the strong client demand for such products.
- Periodic occurrence of market events, such as major drops in equity markets or sudden jumps in volatility, contribute to what we may call an implied volatility bias. Traders cover the risk related to these events by charging higher option prices leading to higher implied volatilities. Historically, whenever a sudden fall in equity markets occurred, the market overestimated the probability of re-occurrence of such an event.

Search for opportunities: Equity Volatility arbitrage

- **Case S&P 500:** The realized monthly volatility (RVOL) versus the implied 1m volatility (Strike SPX) computed via the option market. Note the RVOL spikes in Nov 1997, Sep 1998, and during the equity bear period in 2002.



Search for opportunities: Equity Volatility arbitrage

- Expected variance in the risk-neutral measure (in the simplified case where interest rates = 0)

$$\text{Exp variance} = (\text{Strike})^2 = \int dk\ \theta(S-k)\ P(k)/k^2 + \int dk\ \theta(k-S)\ C(k)/k^2$$

where S is the stock's present value, P(k) and C(k) respectively the price of a put and a call of strike k, and θ(x) = 1 for x>0, θ(x) = 0 otherwise. Implied volatility here will be understood as the square root of the variance.

- Realized volatility

$$RVOL_m = 100 \times \sqrt{\frac{\sum_{i=1}^{N}\left[\ln\left(S_i / S_{i-1}\right)\right]^2}{n}} \times \sqrt{252}$$

with

S_i = The daily Closing Level of the relevant Index (except for the expiry day) on day i

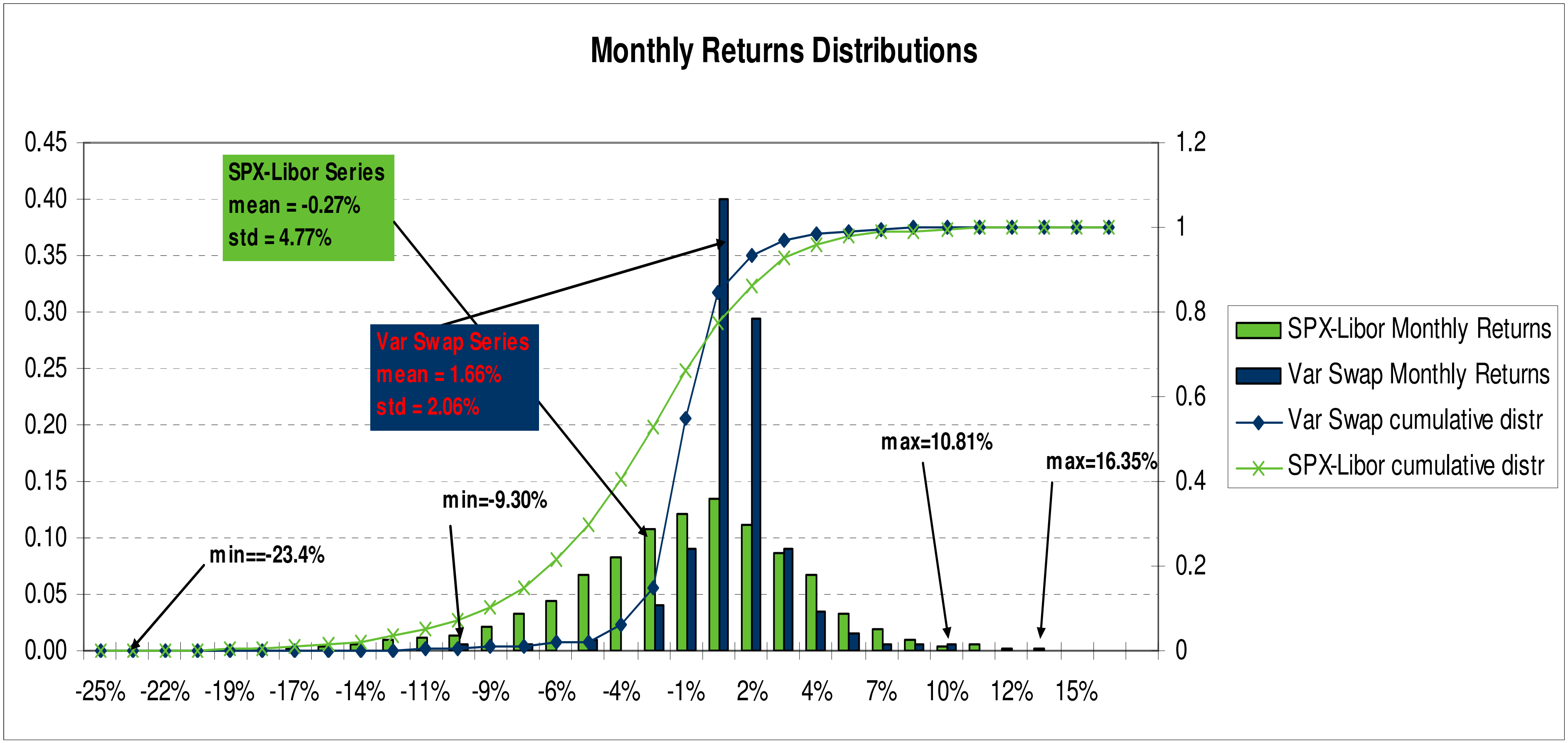
n = Number of Business Days in Period from Rolling to Expiry Date

N = n - Disrupted Days

- Variance swap payoff at maturity ~ $(\text{Strike})^2 - \text{RVOL}^2$

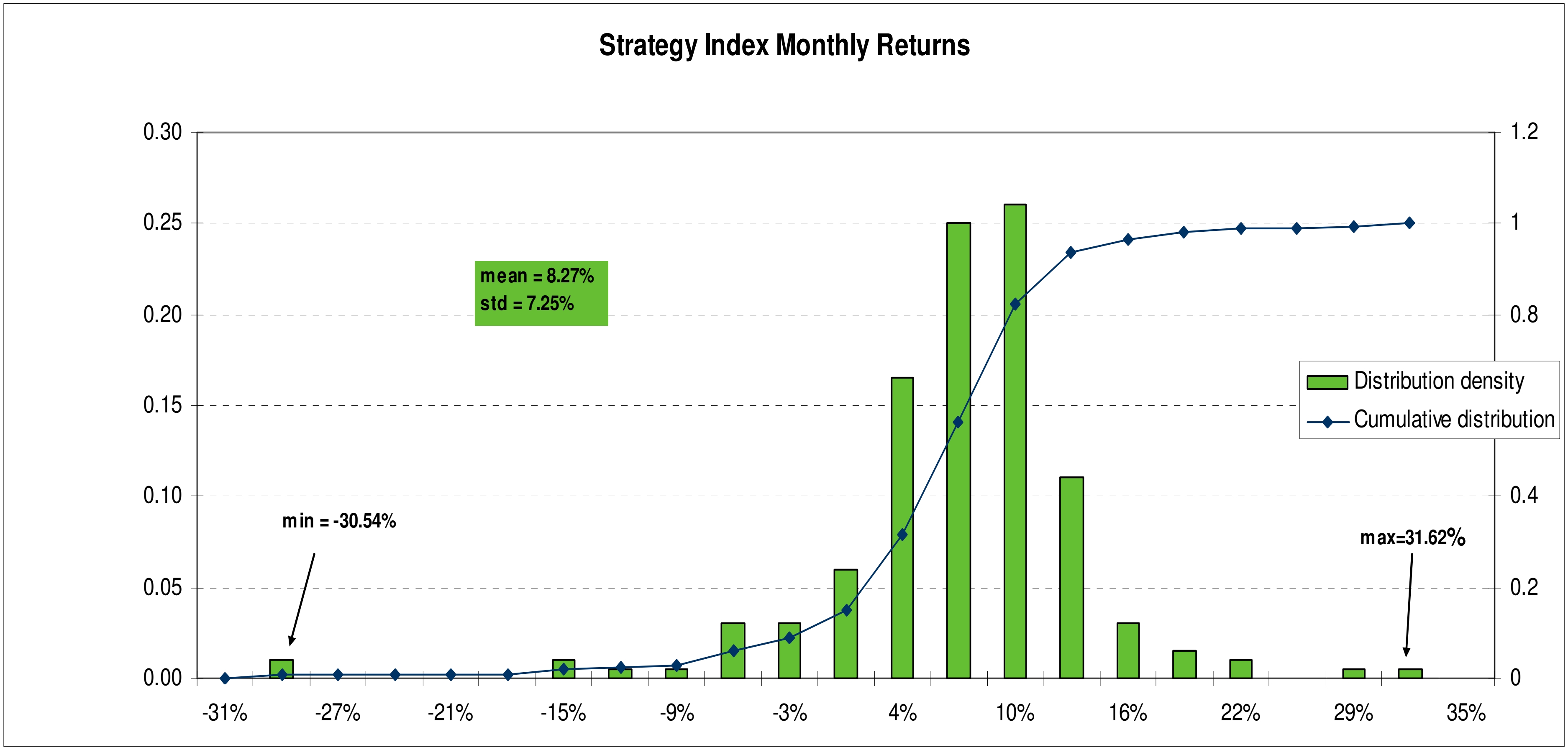
Search for opportunities: Equity Volatility arbitrage

- "Alpha" type: Distribution of returns of 1 month S&P 500 variance swaps inferred from historical data (Jan 1990 – Sep 2006).
- Typical "Beta" type: Returns of 1 month investments in the S&P 500 financed at the LIBOR rate are shown for comparison.



Search for opportunities: Equity Volatility arbitrage

- A Dresdner Kleinwort proprietary strategy index built using a gap risk controlling, implied volatility dependent leverage function. Note that in this graph transaction costs are not taken into account.



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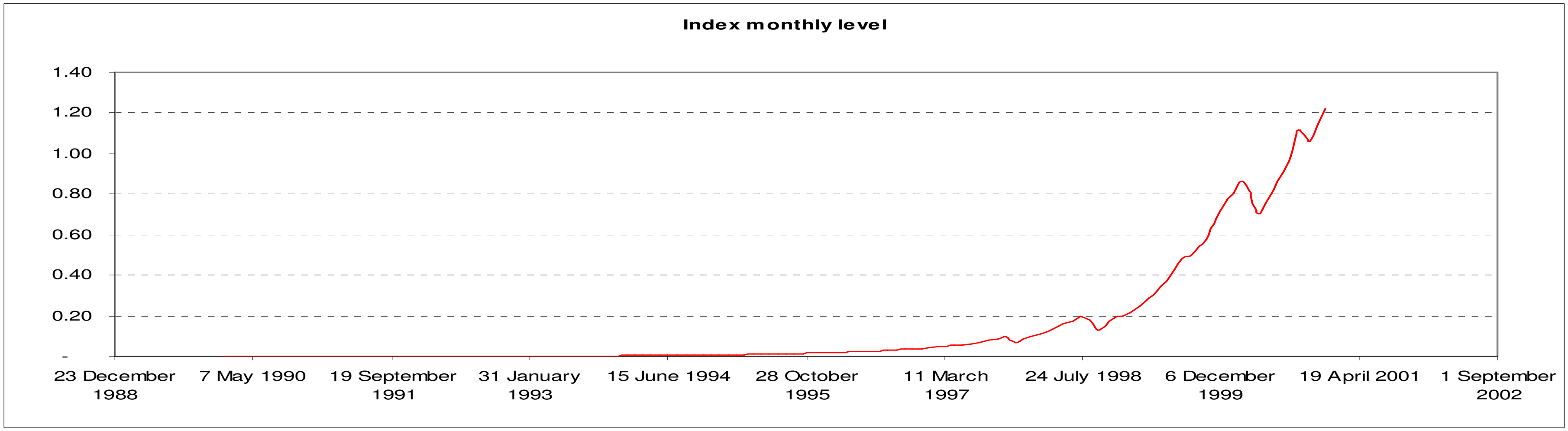
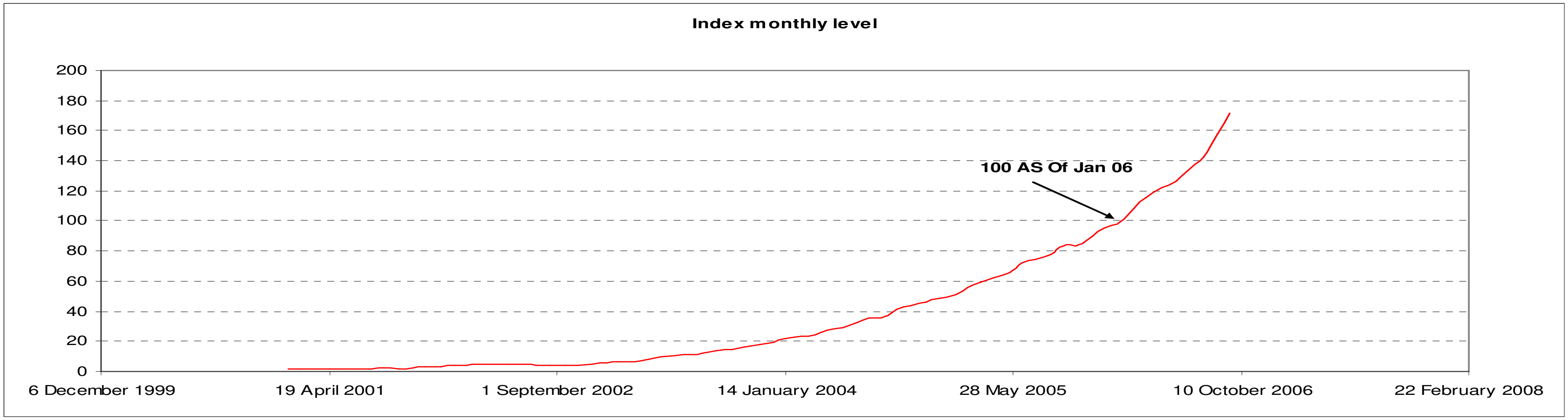
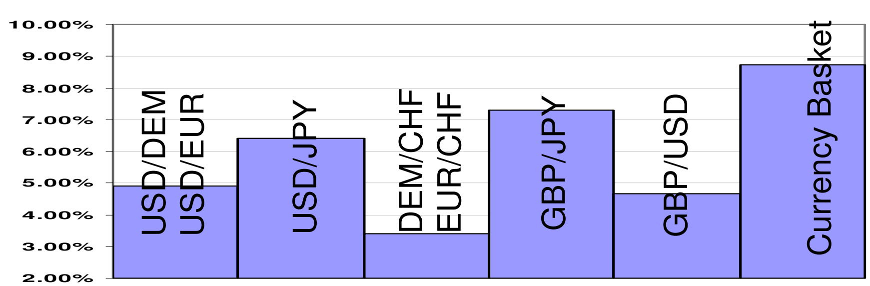




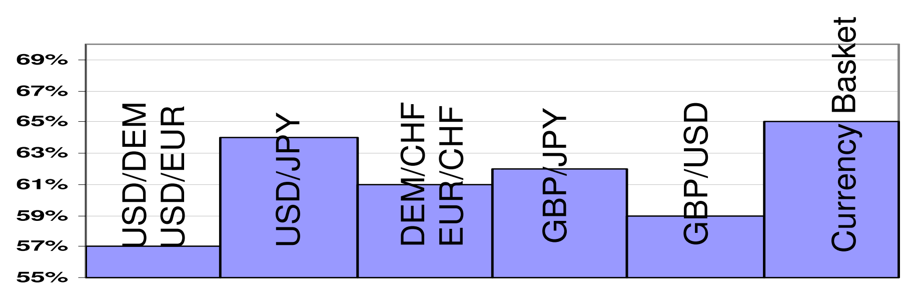
Search for opportunities: FX forwards arbitrage

- The forward FX rate between low and high interest rate currencies tends to over-predict the future realised spot FX rate (Forward FX Bias).
- FX Forward rates are mainly used to hedge exchange rate risk. If market makers are risk-neutral, the FX Forward rate is a proxy for the expected future FX Spot rate. Interest rate differentials mainly reflect expected inflation differentials.
- When a country assumes tighter domestic monetary policy (i.e. raise domestic nominal interest rates), usually the currency appreciates on the spot market and is expected to depreciate in the future – as implied by the FX Forward.
- Back-testing on historical data between 1995 and 2005 (for a basket of the 5 Major OECD Currencies USD, GBP, JPY, EUR, CHF) shows that the one-month FX Forward overestimates the one-month spot rate – based on individual pairs – between 57% and 64% of cases and the average annualised overestimation stands between 3.5% and 7.3%.
- Choosing the currency pair with the widest interest rate differential within the above basket produces the highest percentage of cases where the one-month FX Forward rate overestimated the FX spot rate at 65% and the highest overestimation level at 8.8%, respectively.

Annualized overestimation of the FX forward with respect to realized FX Spot for different currency pairs.



Historical Percentage that the FX Forward over estimated the FX Spot



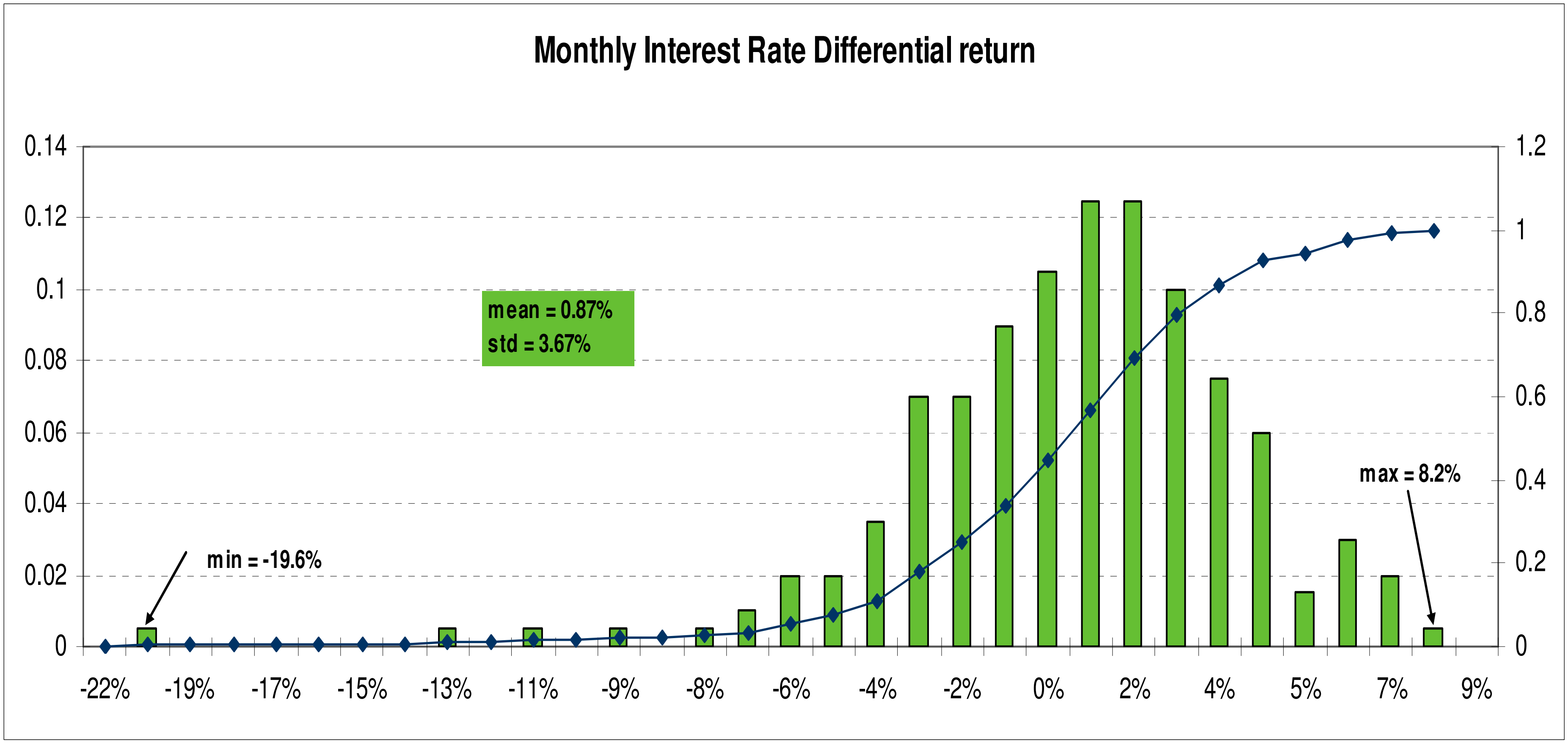
Search for opportunities: FX forwards arbitrage

- Let FX_H and FX_L be the exchange rates of the domestic currency (USD) with respect to the highest and lowest rate currencies in the basket USD, EUR, CHF, GBP, JPY.
- Lending at the highest rate and borrowing at the lowest between t and t+Δt gives the following

$$\text{Payout at } t + \Delta t = \left(\frac{FX_H(t) \times (1 + r_H \Delta t)}{FX_H(t + \Delta t)} - \frac{FX_L(t) \times (1 + r_L \Delta t)}{FX_L(t + \Delta t)} \right)$$

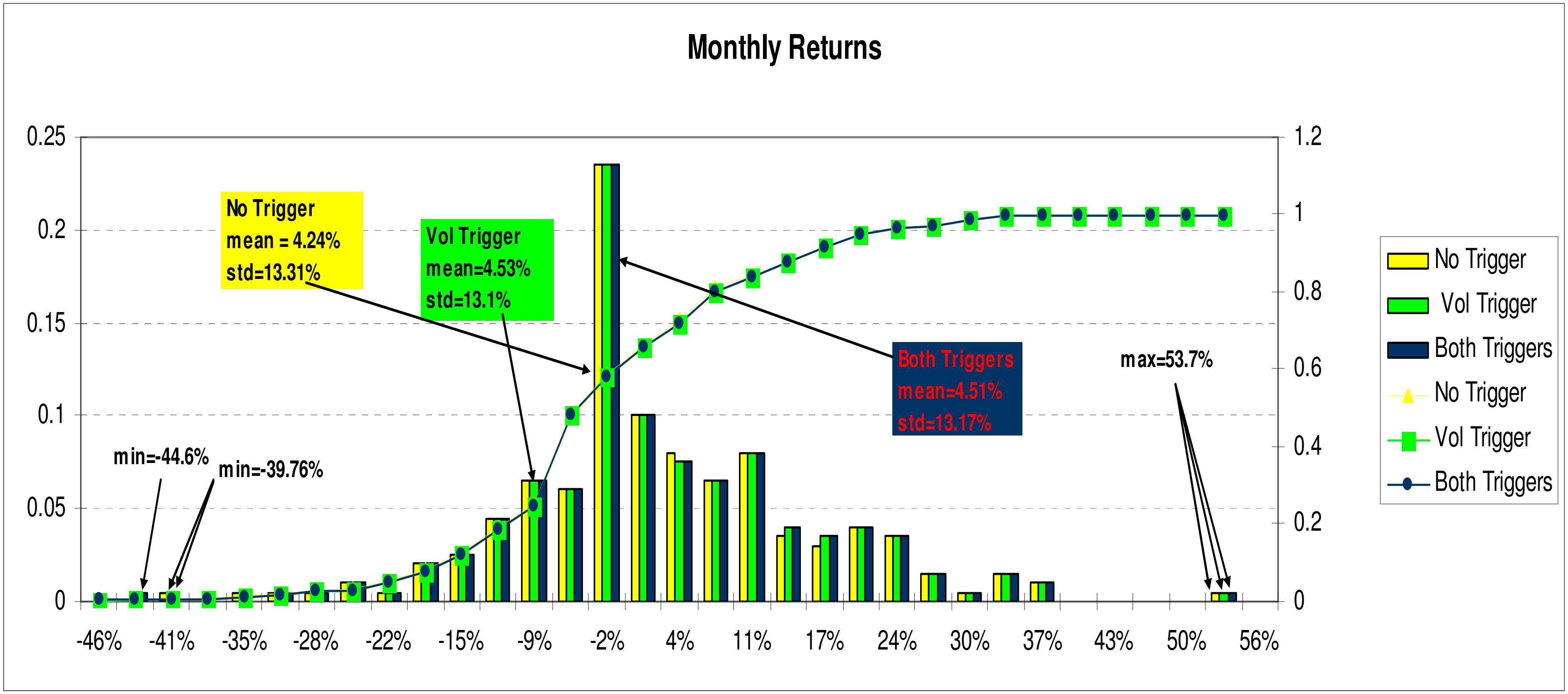
Search for opportunities: FX forwards arbitrage

- "Alpha" type: Distribution of returns of 1 month contracts where we lend at the highest rate and borrow at the lowest rate (highest and lowest are determined monthly within a fixed basket of major currencies). The chosen base currency is USD. Data of Jan 1990- Oct 2006.



Search for opportunities: FX forwards arbitrage

- Strategy indices built using gap risk controlling leverage functions with or without added controls. A volatility trigger prevents from entering the monthly investment roll in presence of too high implied FX volatilities. An additional trigger prompts unwinding of the investment whenever a given stop loss limit is breached. Transaction costs are here not taken into account.



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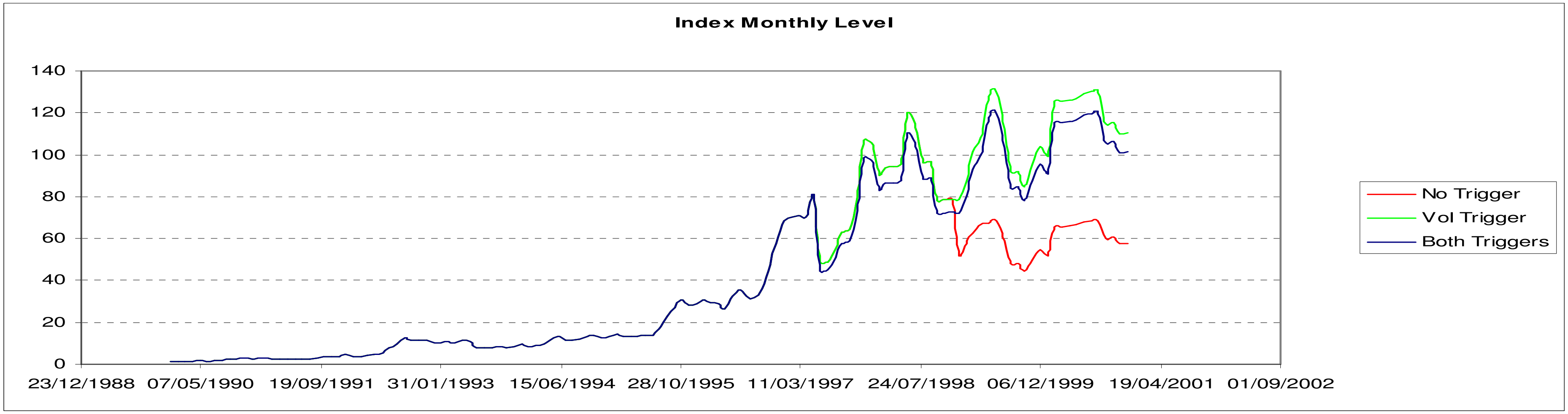
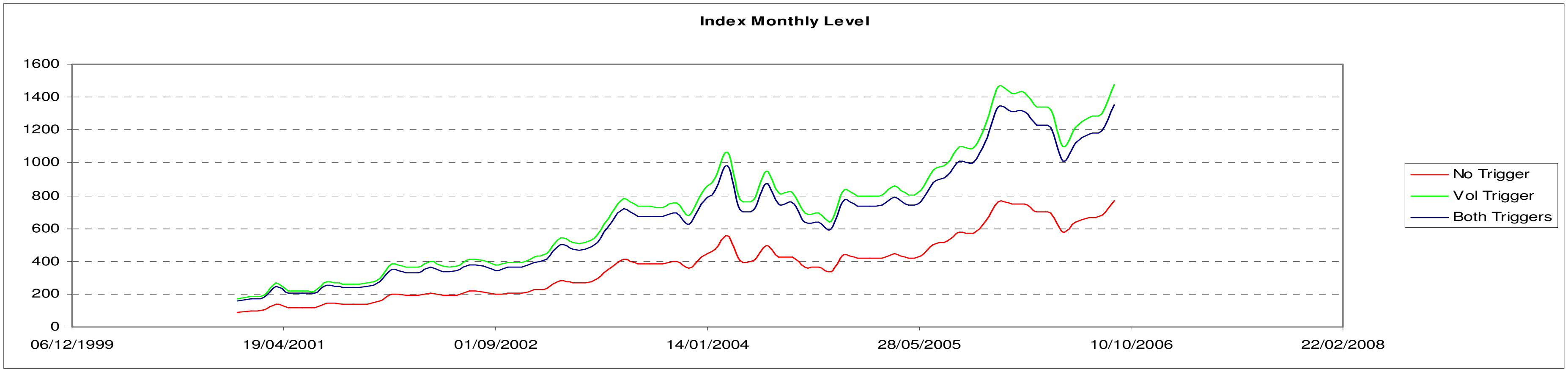




Search for opportunities: FX forwards arbitrage

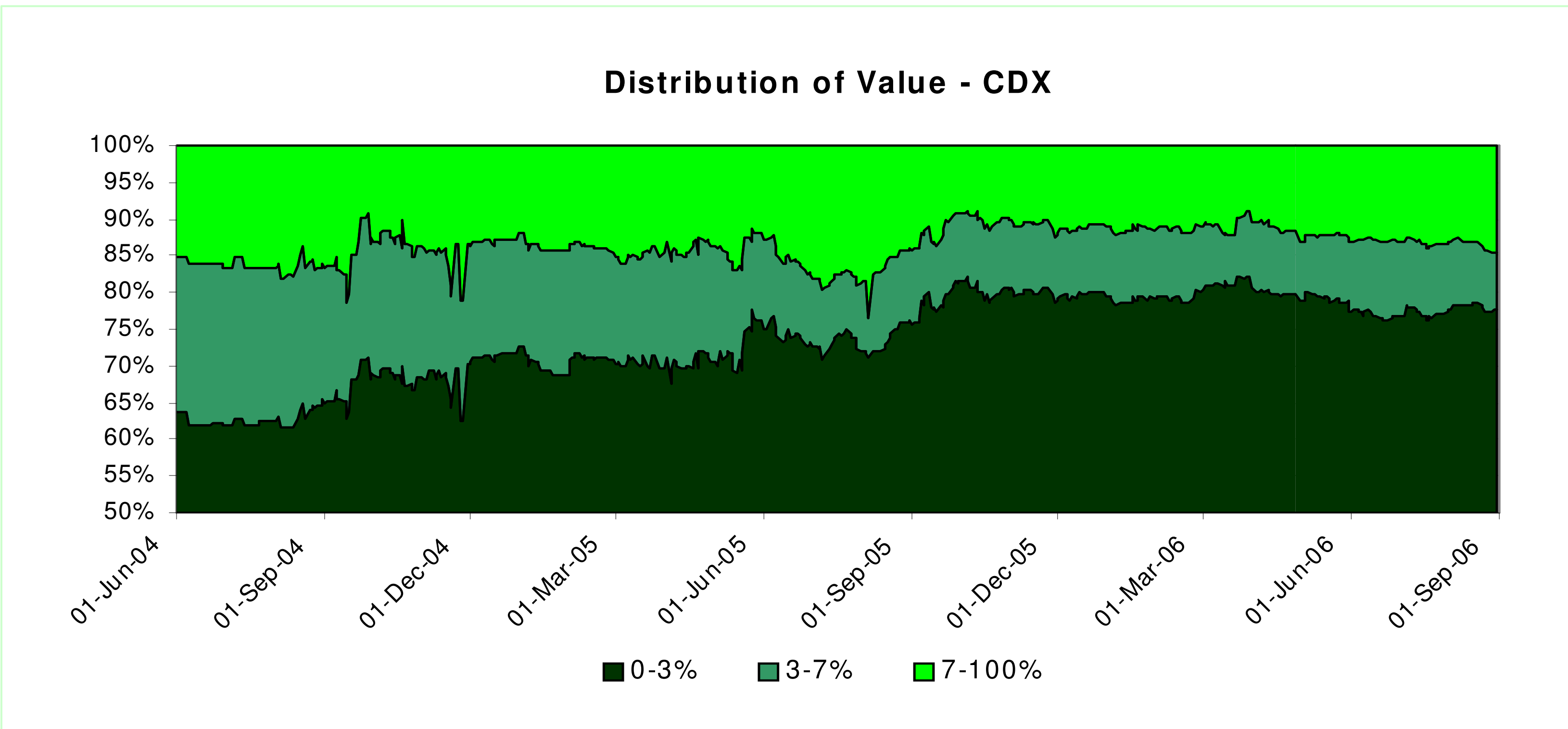
- The addition of more sophisticated opportunity weighting leverage functions can substantially improve the index. Represented below is the result obtained via a leverage function which also depends both on past trend and on FX volatility skew (DIDA-M Index).

Search for opportunities: Relative value of CDO tranches

- Analysis below shows the premiums paid to protection sellers for various parts of the tranched 5 year CDX IG index, expressed as a percentage of the premium on the un-tranched index. That is for the tranche [x,y], the ratio

$$(\int dp(l)\ \theta(l-x)\ \theta(y-l)\ l)\ /\ (\int dp(l)\ l),$$ where l is the total percentage loss in the portfolio between time 0 and 5 years

- As of Q3 2005, strong fundamentals in conjunction with technical factors have resulted in a re-distribution of value from mezzanine to equity tranches



Search for opportunities: Relative value of CDO tranches

- Rolling position of the form

$$\Pi = \text{Equity}(0\text{-}3\%) - \Delta\ \text{Mezzanine}(0\text{-}7\%)$$

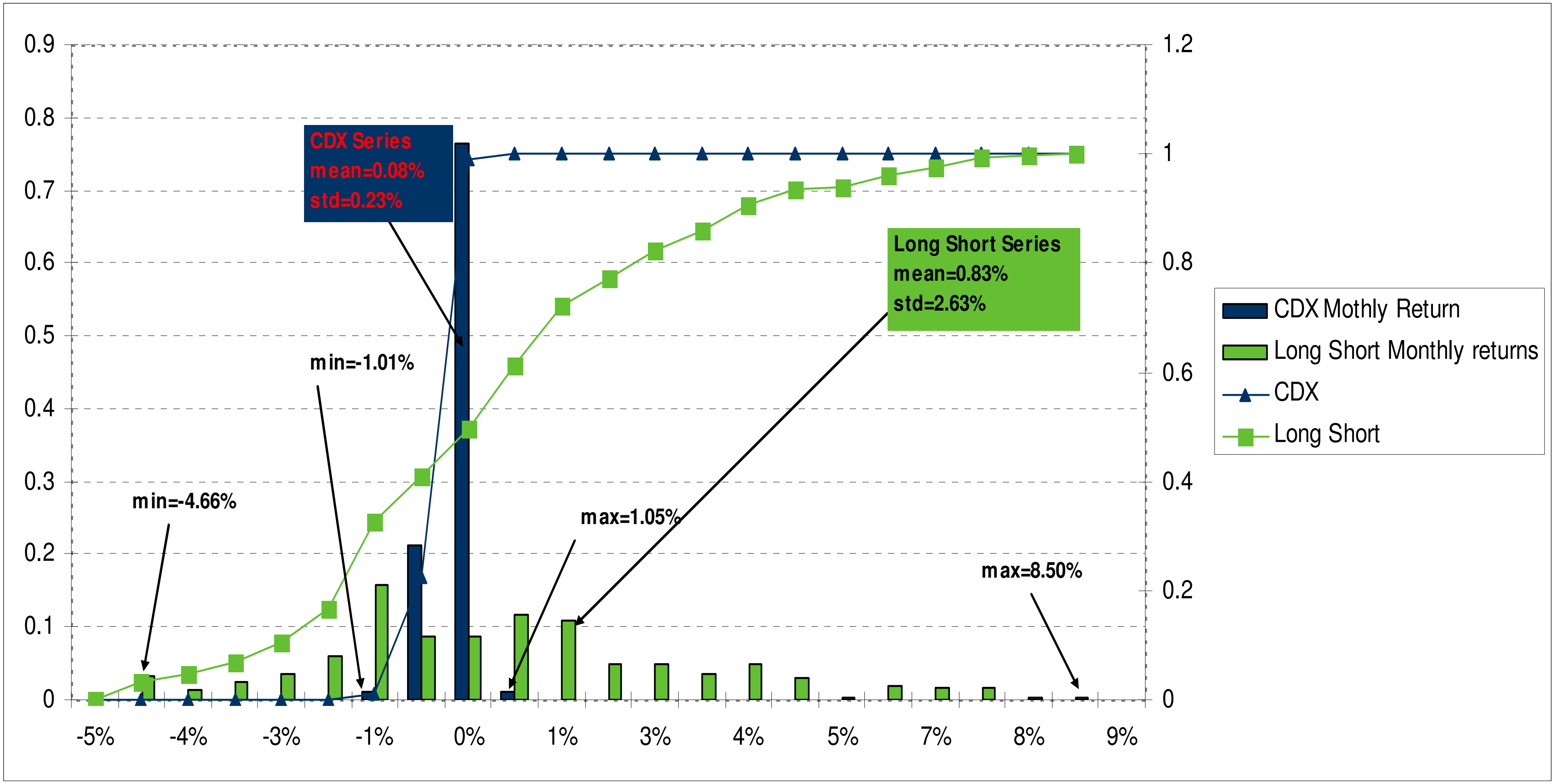
such that for a parallel market move, i.e.,

$$s \rightarrow s + \delta s \text{ in all spreads in the CDX index}$$

we have $\delta\Pi = 0$.

Search for opportunities: Relative value of CDO tranches

- Distribution of monthly returns of a (long 0-3% equity, short 3-7% mezzanine) delta neutral position in CDX tranches. Data of June 2005 – Sep 2006.
- As a reference the distribution of monthly returns of a CDX long positions is also shown. Data of June 2004 – Aug 2006.



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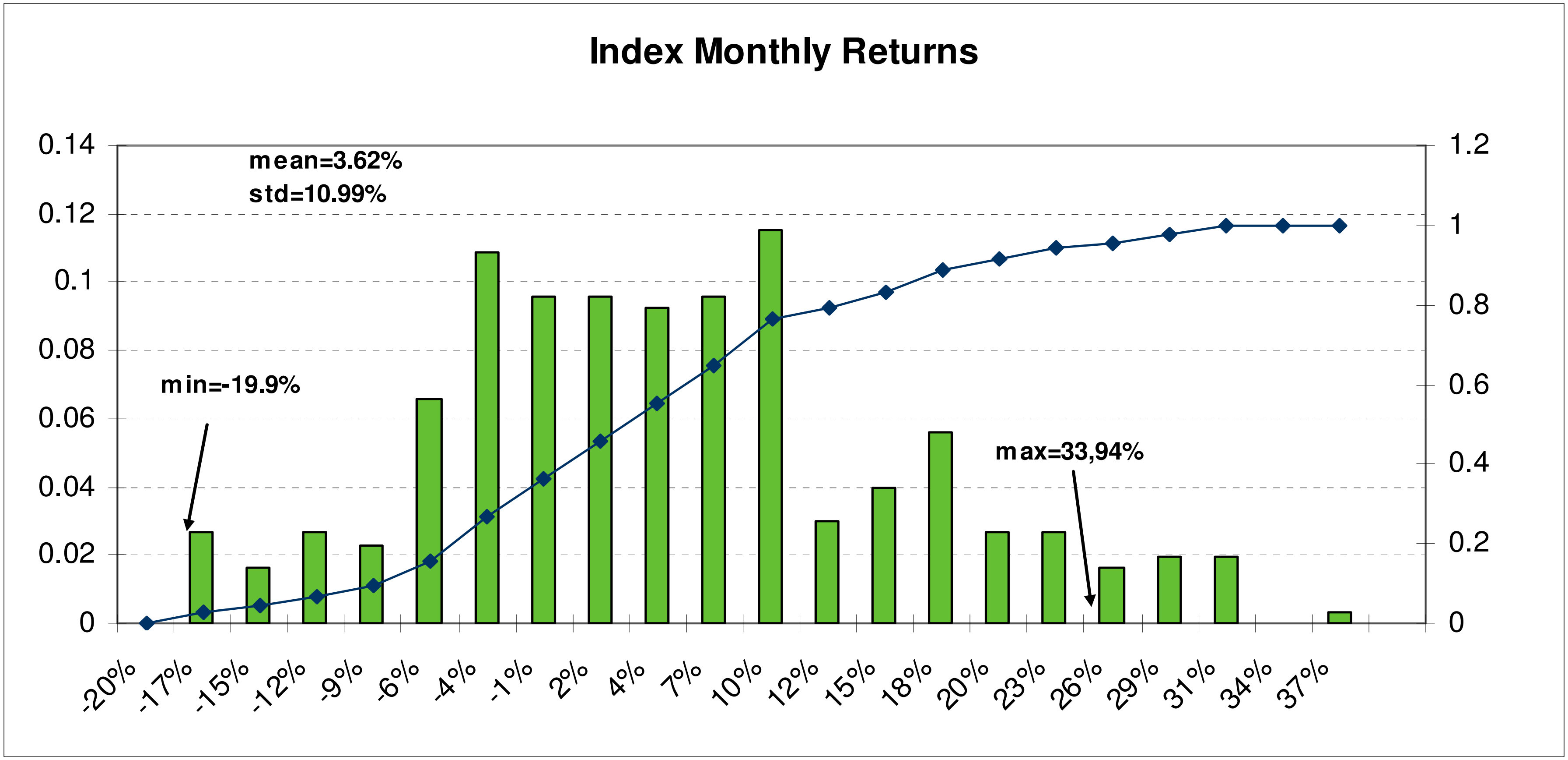
Search for opportunities: Relative value of CDO tranches

- Strategy index built using gap risk controlling leverage function λ = 4. Transaction costs are here not taken into account.



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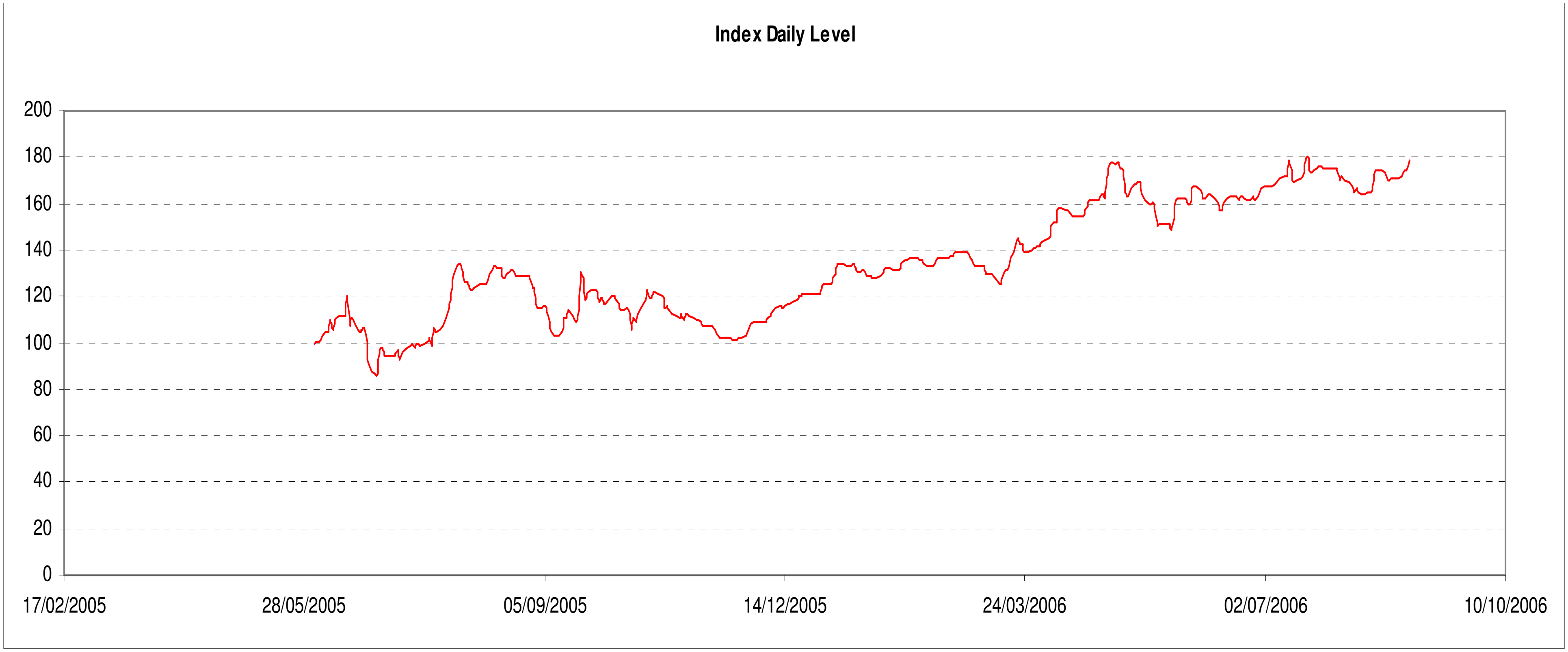

Strategy index derivatives

Pricing the strategy index and transaction costs

- If we could ignore liquidity, and transaction costs issues, the value of a strategy index as defined before would obviously by construction be equal to the initial cash infusion C(0).
- Consider a simple cash settled contract according to which the index buyer will receive the index value I(T) at time T. If liquidity of the underlying rolling investment portfolio were unlimited, and bid/offer spreads equal to zero, the present value of the contract would be C(0) = 100. Given the great number of derivatives contracts involved, and their potentially large amounts, in fact the price at which the contract can be sold has to be considerably larger.
- Note that transaction costs are strongly dependent on the path of the index value from time 0 to time T.
- The index seller could choose to replicate rather than actually enter the derivative contracts in the rolling underlying portfolio. Bid/offer costs of the derivatives would thus resurface in the form of replication costs.
- The problem of pricing the index falls beyond the scope of risk neutral evaluation; it has nevertheless a strong impact on price competition among banks and on internal risk management policy.

Strategy index derivatives

Elementary derivatives: main examples

Here we list just two basic examples. Note that path-dependency is a built in intrinsic feature of the index.

1. Standard strategy index derivatives guaranteeing a minimum payout are built with payoffs of the form

$$\mathrm{Max}[\ I(T),\ K\ ]$$

without or with barrier features.

2. Partially guaranteed funds. Let

$$\mathrm{Max}_i = \mathrm{Max}\{\ I(T_j)\ \big|\ j=0,\dots,\ i\ \}.$$

If k is number between 0 and 1, a fund with partial guarantee which can be withdrawn at any time T_n pays at T_n the cash amount

$$\mathrm{Max}[\ I(T_n),\ k \cdot \mathrm{Max}_{n-1}]$$

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