Model risk and operational risk

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Basel II

Basel Committee
on Banking Supervision

International Convergence of Capital Measurement and Capital Standards

A Revised Framework
Comprehensive Version

This document is a compilation of the June 2004 Basel II Framework, the elements of the 1988 Accord that were not revised during the Basel II process, the 1996 Amendment to the Capital Accord to Incorporate Market Risks, and the 2006 paper on the Application of Basel II to Trading Activities and the Treatment of Double Default Effects. No new elements have been introduced in this compilation.

June 2006

BANK FOR INTERNATIONAL SETTLEMENTS

http://www.bis.org/publ/bcbs128.htm

The three pillars approach

- **First pillar**: **Minimum Capital Requirements** (quantification of risk)
  - Specifies the guiding principles for the estimation of regulatory/economic capital.
  - Operational risk is included as a new type of risk.
- **Second pillar: Supervisory Review Process.**
- **Third pillar: Market Discipline (+ public disclosure)**

Operational Risk: Definition

Losses from events that cannot be considered as market or credit risk events.

(source: Basel II)

644. Operational risk is defined as the risk of loss resulting from **inadequate** or **failed internal processes**, **people** and **systems** or from **external events**.

This definition includes legal risk, but excludes strategic and reputational risk.

Operational Risk: Measurement

Measurement Approaches

- Basic Indicator Approach

$$K_{\text{BIA}} = \alpha \times \text{EI}, \text{ where } \begin{cases} \alpha = 0.15 \\ \text{EI=gross income (mean of the last 3 years)} \end{cases}$$

- Standardised Approach.

$$K_{\text{TSA}} = \sum_{i=1}^{8} \beta_i \times \text{EI}_i, \text{ where } \begin{cases} \beta_{\text{i}} \text{ are defined by the regulator} \\ \text{EI}_{\text{i}} \text{ are the gross income for line i.} \end{cases}$$

- Advanced Measurement Approaches (AMA)
  - Scorecard approach.
  - Loss Distribution approach.

AMA Soundness Standard (Basel II)

667. Given the continuing evolution of analytical approaches for operational risk, the Committee is not specifying the approach or distributional assumptions used to generate the operational risk measure for regulatory capital purposes. However, a bank must be able to demonstrate that its approach captures potentially severe 'tail' loss events. Whatever approach is used, a bank must demonstrate that its operational risk measure meets a soundness standard comparable to that of the internal ratings-based approach for credit risk, (i.e. comparable to a **one year holding** period and a **99.9th** percentile confidence interval).

Business lines & risk types

	Risk type						
Business line ↓	Internal fraud	External fraud	Employment Practices and Workplace Safety	Clients, Products & Business Practices	Damage to physical assets	Business disruption and system failures	Execution, Delivery & Process Management
Corporate Finance							
Trading & Sales							
Retail Banking							
Commercial Banking							
Payment and Settlement							
Agency Services and Custody							
Asset Management							
Retail Brokerage							

Loss distribution approach

- Model the distribution of the **aggregate losses** for a given **business line & risk type**

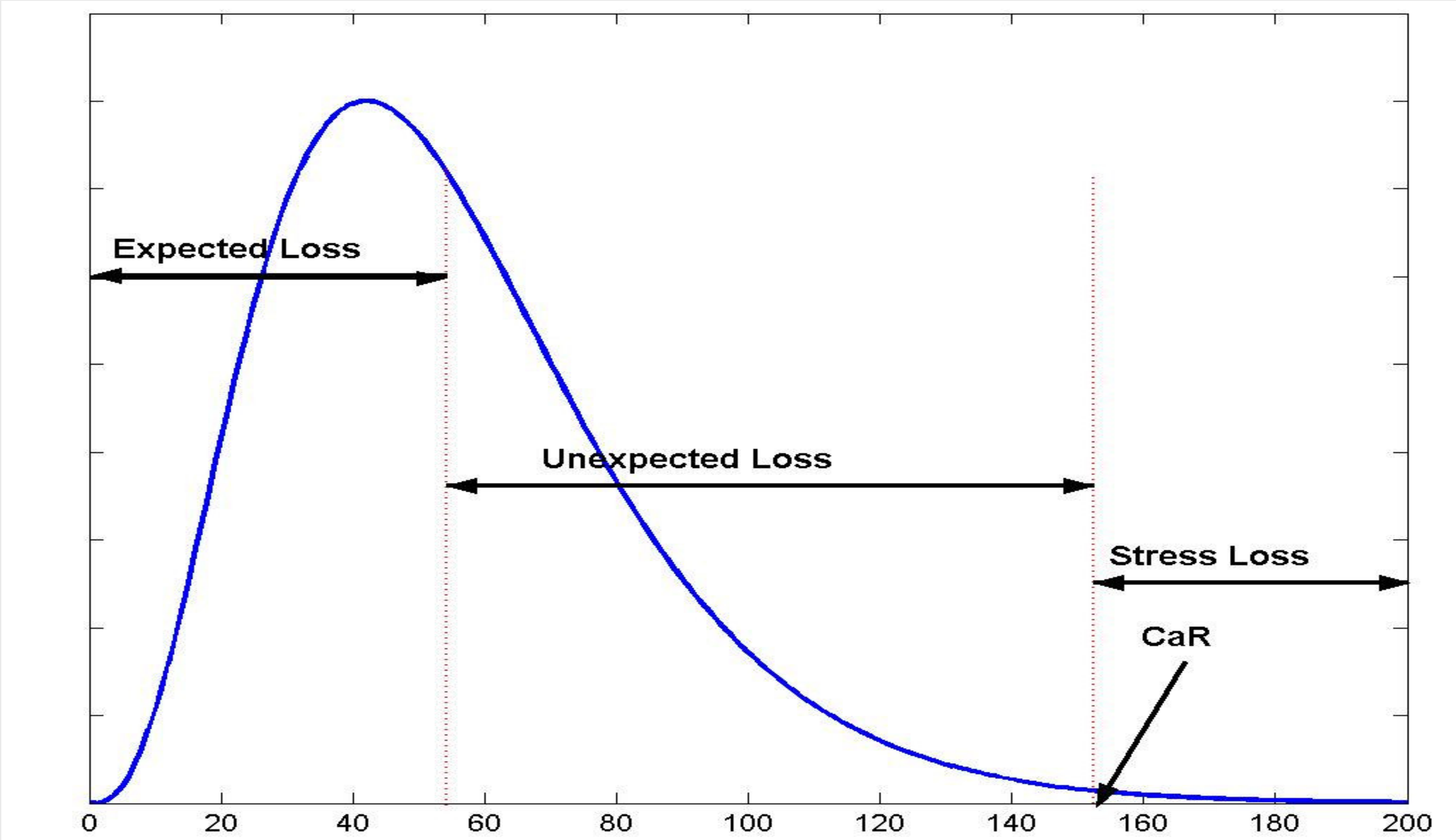
$$Loss_t^{[i,j]} = \sum_{n=1}^{N_t^{[i,j]}} X_{nt}^{[i,j]} \;;$$

$N_t^{[i,j]}$ is the number of losses in year t for business line i and risk type j.

- Calculate **Capital at Risk** (99.9% percentile) of the aggregate loss distribution per **business line & risk type** and **add** them.

$$CaR = \sum_{i=1}^{8} \sum_{j=1}^{7} CaR^{[i,j]}$$

Expected & unexpected loss



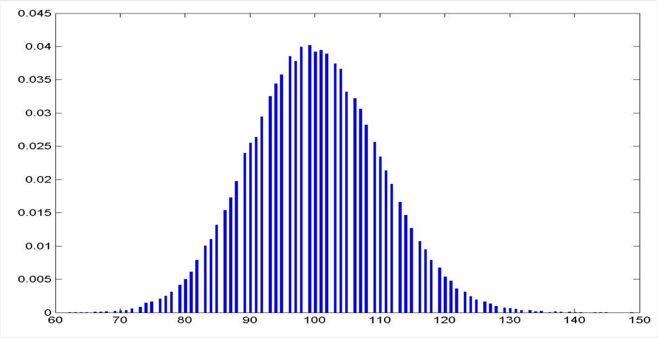
Actuarial models: Frequency + Severity

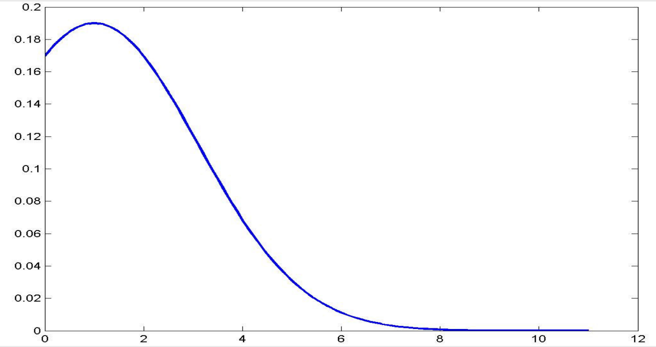
- Hypothesis
  - **Severities** of losses are **independent**
  - **Severities and frequencies** are **independent**
- Model separately
  - Frequency $\{N_t\}$

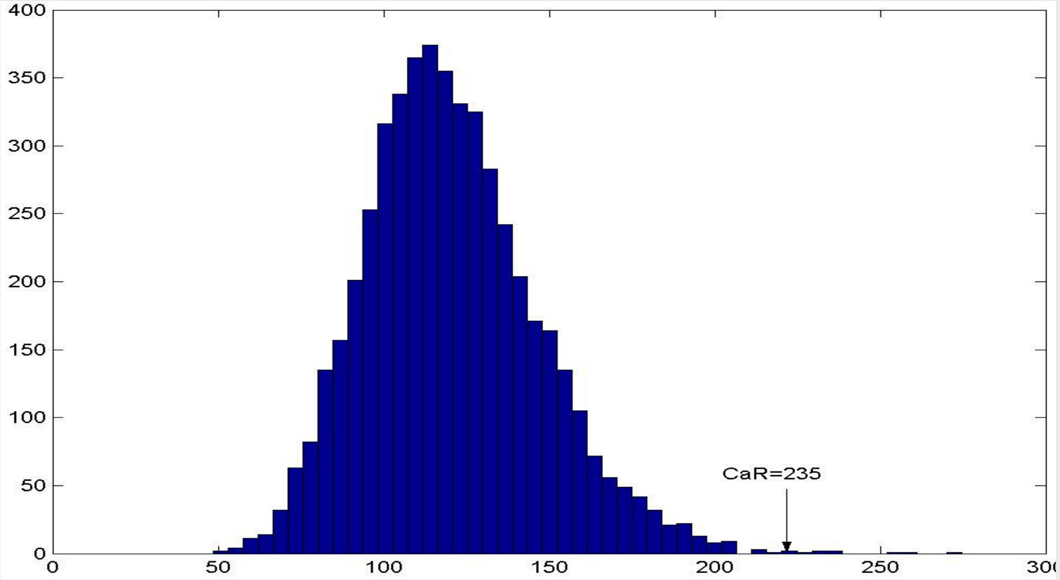
    E.g. Poisson, negative binomial, Cox process,…
  - Severity $\{X_{nt}\}$

    E. g. Log-normal, Gaussian inverse, Weibull, …
- Obtain the aggregate loss distribution by combining these distributions. [Panjer, FFT, **MC Simulation**]

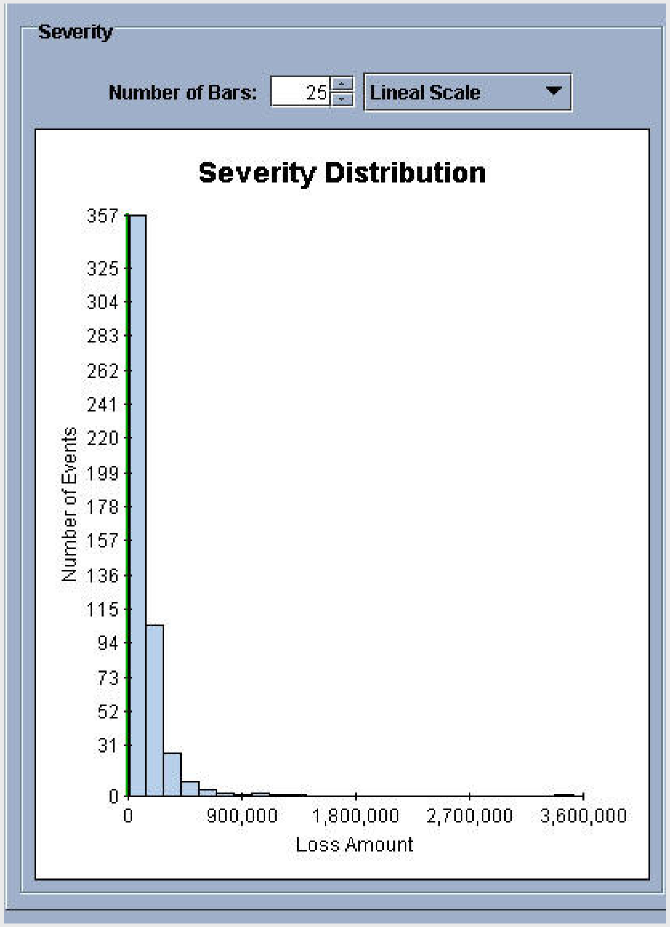
Aggregation of frequency and severity dists.

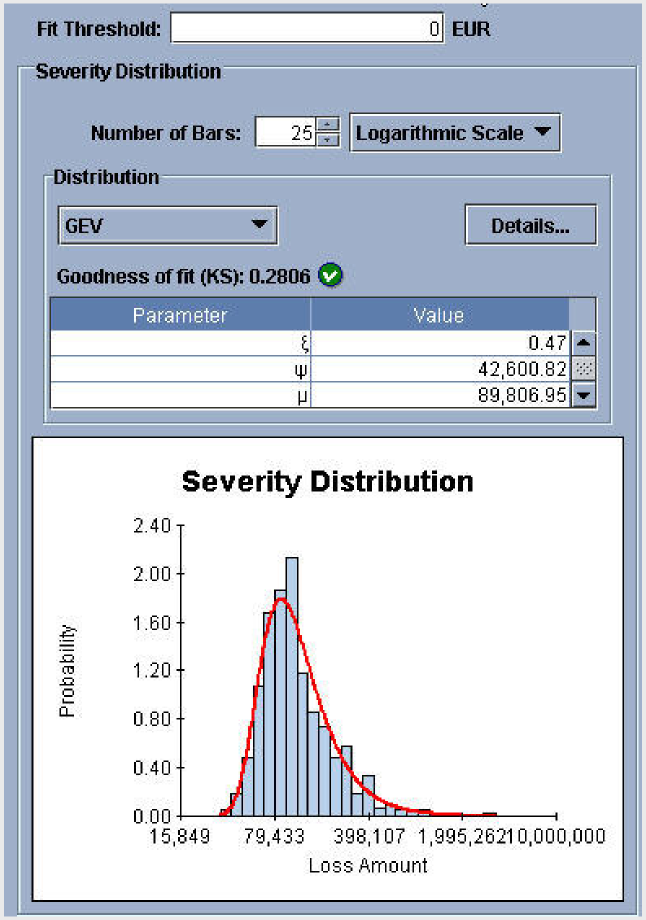






Heavy tails in the severity distribution

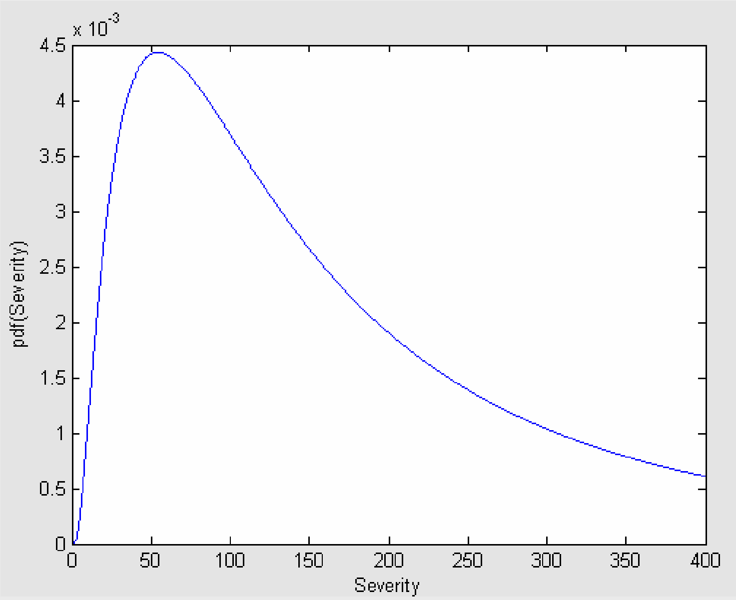




The Lognormal Distribution

- Probability density function

$$LN(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma}\,x}\exp\left\{-\frac{(\log x-\mu)^2}{2\sigma^2}\right\}$$



Extreme value theory and operational risk

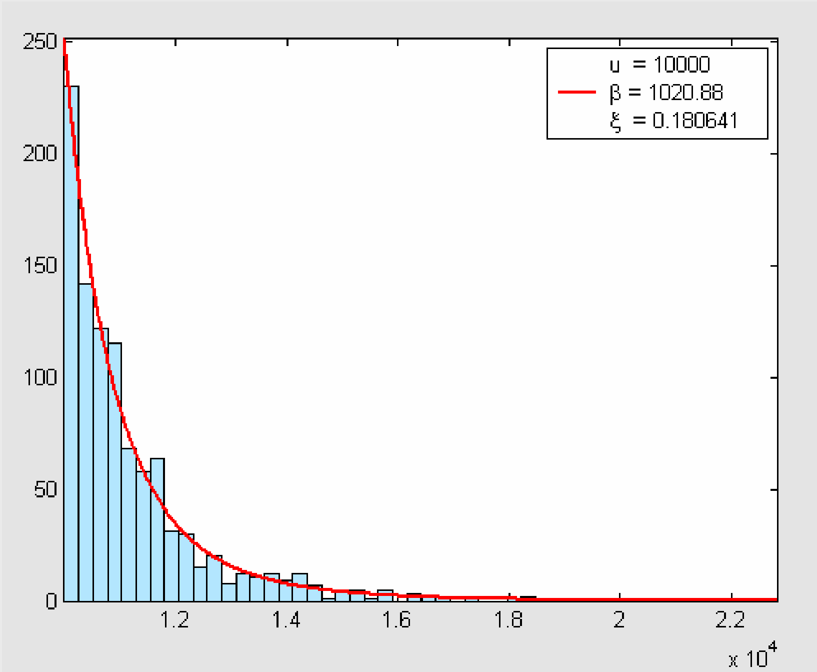
- **Asymptotic regime**:
  CaR is dominated by **single extreme events** from the tail of the severity distribution.
  - Asymptotically, the tail of a distribution is has **Pareto** form.
  - These extreme events should be independent of each other and, assuming stationarity, have a constant probability occurrence per unit time. Hence, the frequency of extreme events should obey a **Poisson** distribution.

- Model: **Poisson + Pareto tail**.

The Generalized Pareto Distribution

- Probability density function

$$GPpdf(x;u,\beta,\xi) = \frac{1}{\beta}\left(1+\frac{\xi}{\beta}(x-u)_+\right)$$

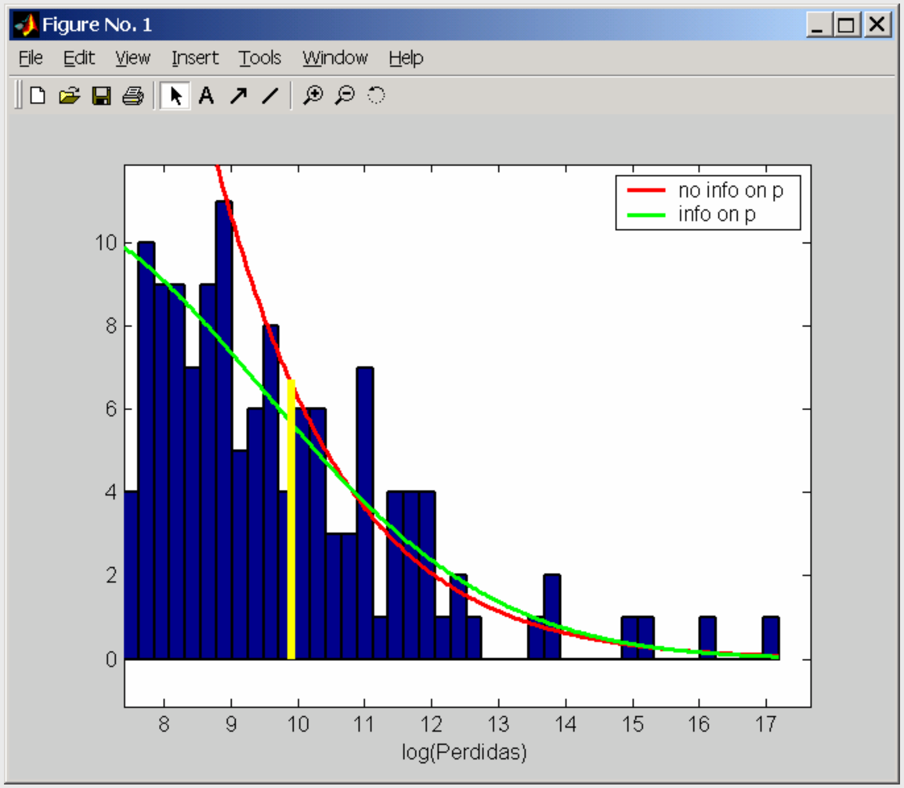


The parameter ξ

- If $\xi \geq 0.5$ the **variance diverges.**
- If $\xi \geq 1$ the **mean diverges.**
  - The **expected loss** is not defined.
  - Empirical estimates of the unexpected loss (the difference between a high percentile of the aggregate loss distribution and the expected loss) **can be negative !**
- In Pareto fits to **empirical** operational loss data, values of ξ **close to 1** and even larger can be found.

A cautionary tale

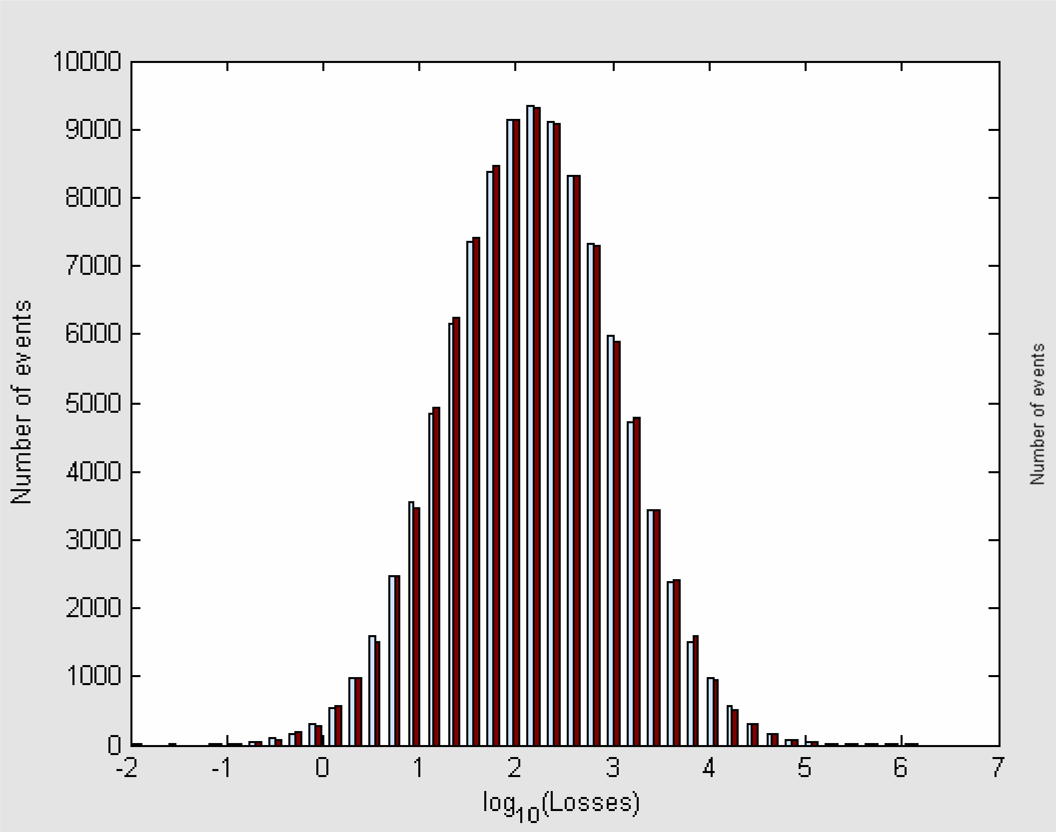
Tails are notoriously **difficult to model**

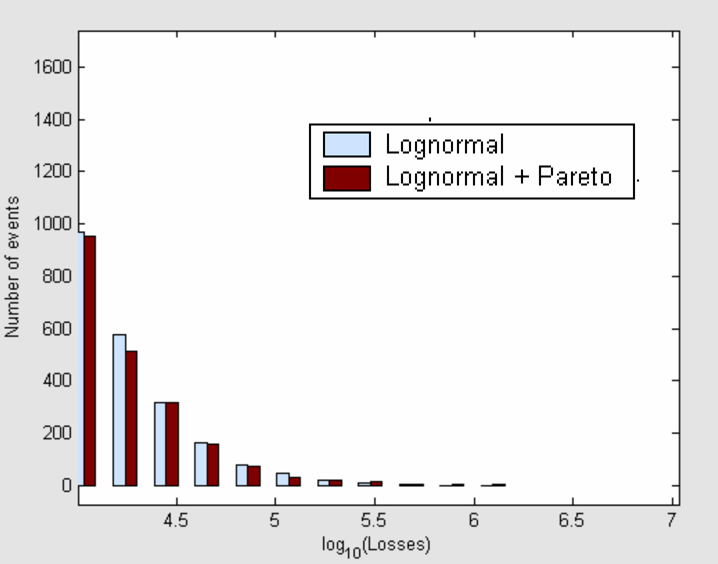


Experiments on simulated data

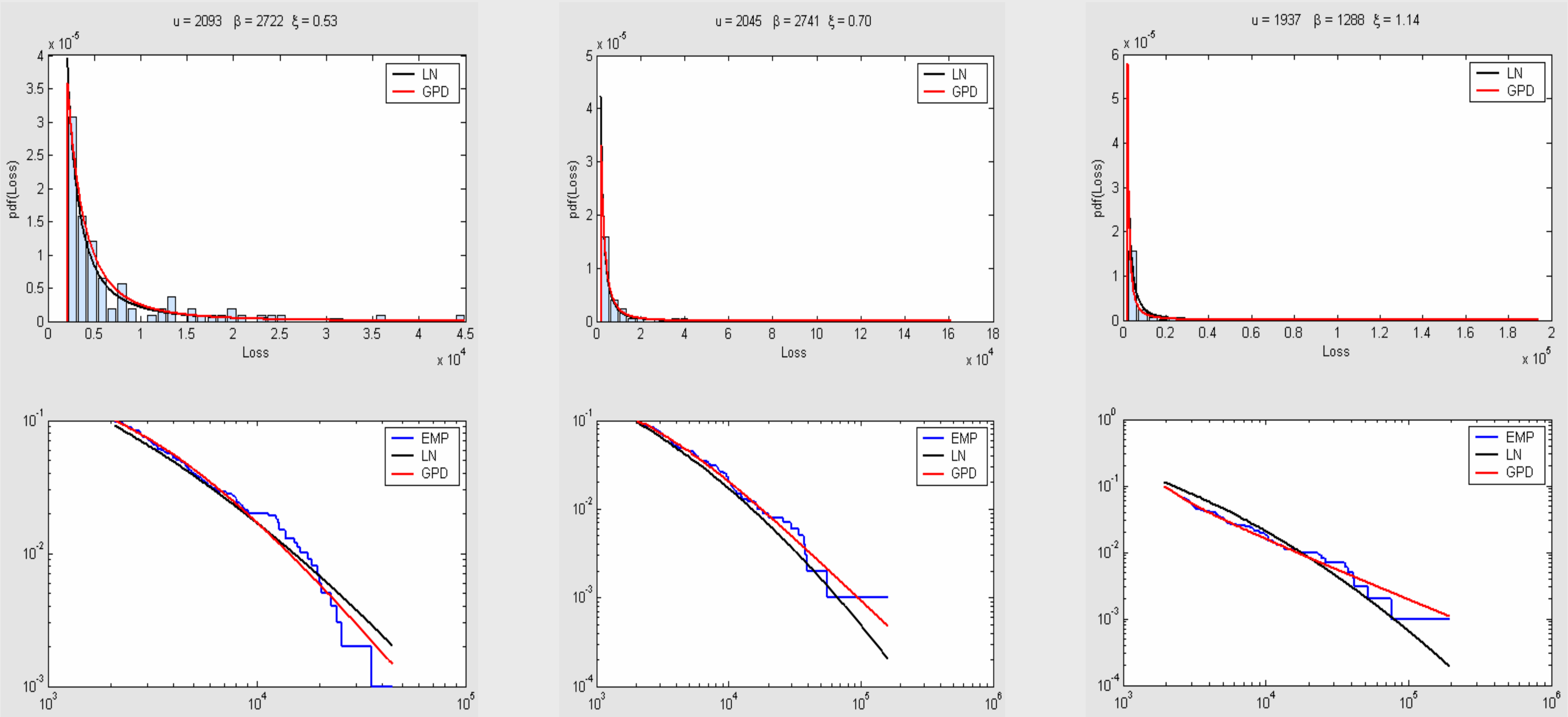
- **5 yeas of data** of losses in operational risk events
- Data sampled from a know distribution:
  - **Lognormal ($\mu = 5, \sigma = 2$)**
  - **lognormal body ($\mu = 5, \sigma = 2$) +**
    Pareto tail ($u = 1930, \beta = 2300, \xi = 0.7$, 10 % of probability in the tail)
- **The sample size is T.**
- Model:
  - Use **Poisson** to model the frequency distribution
    $\lambda = T / 5$
  - Use either **lognormal** or **empirical body + Pareto tail (10%)** to model the severity distribution.

Lognormal vs. Pareto tail (T= 10^5)

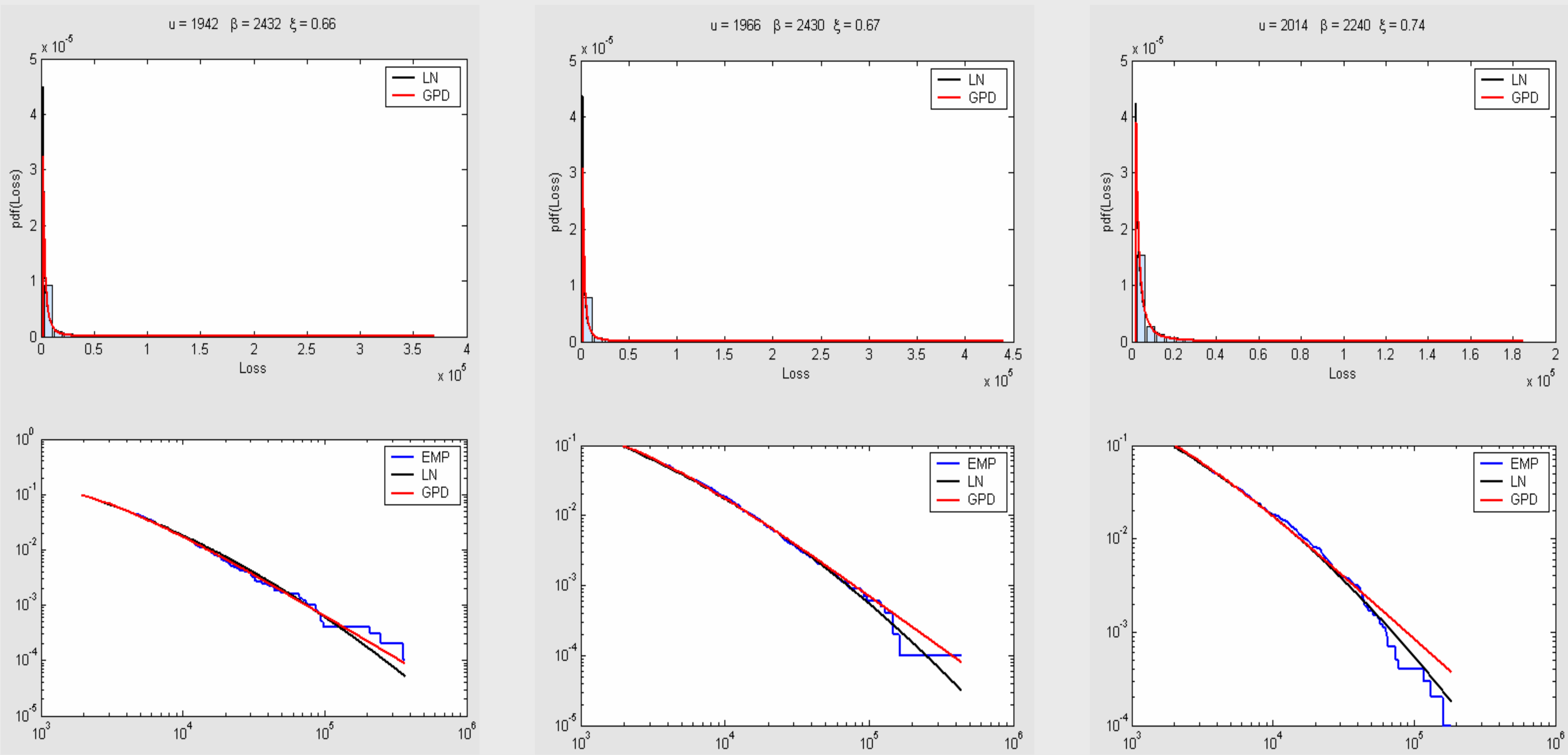




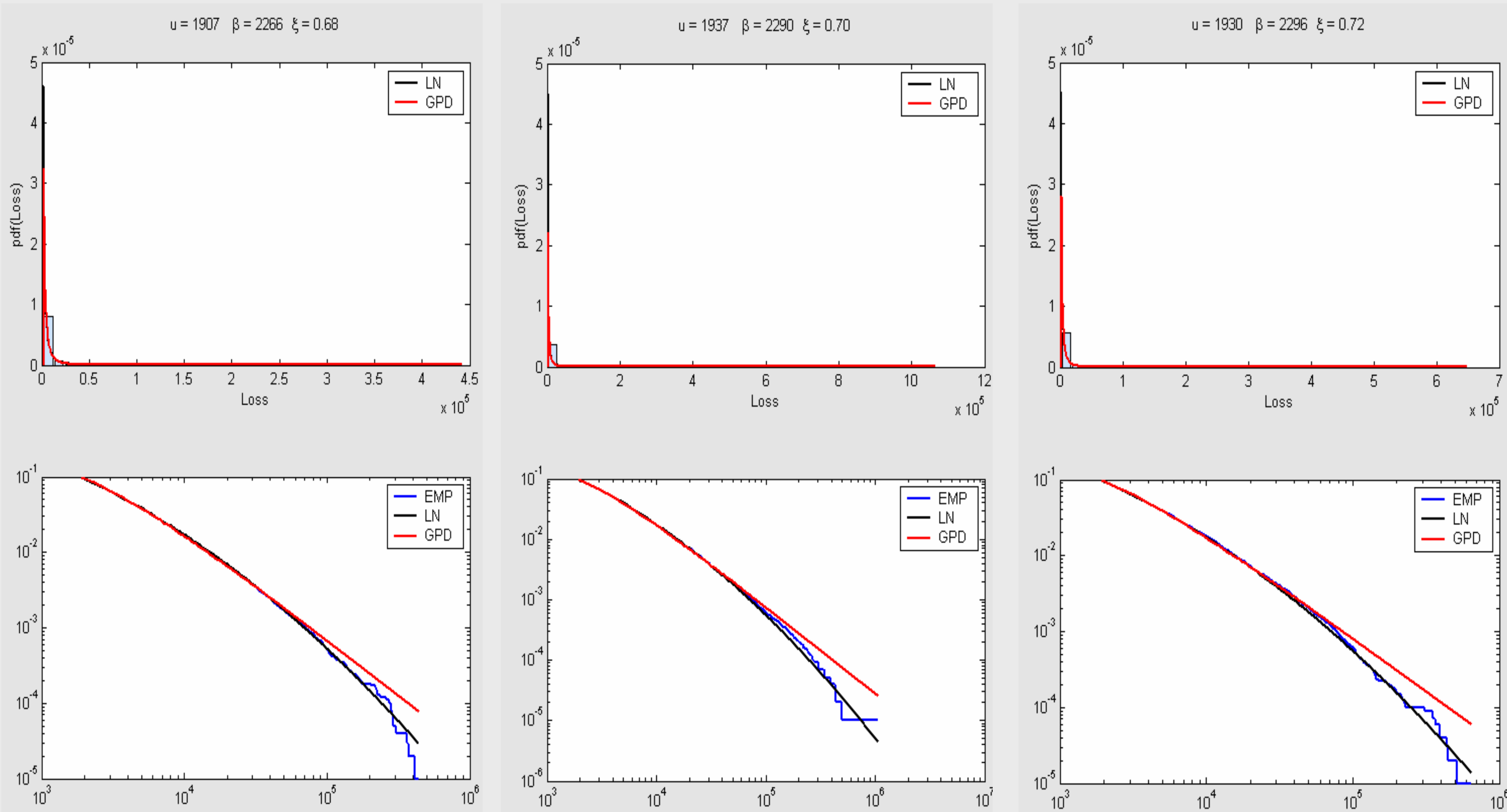
Lognormal data ($T = 10^3$, single simulations)



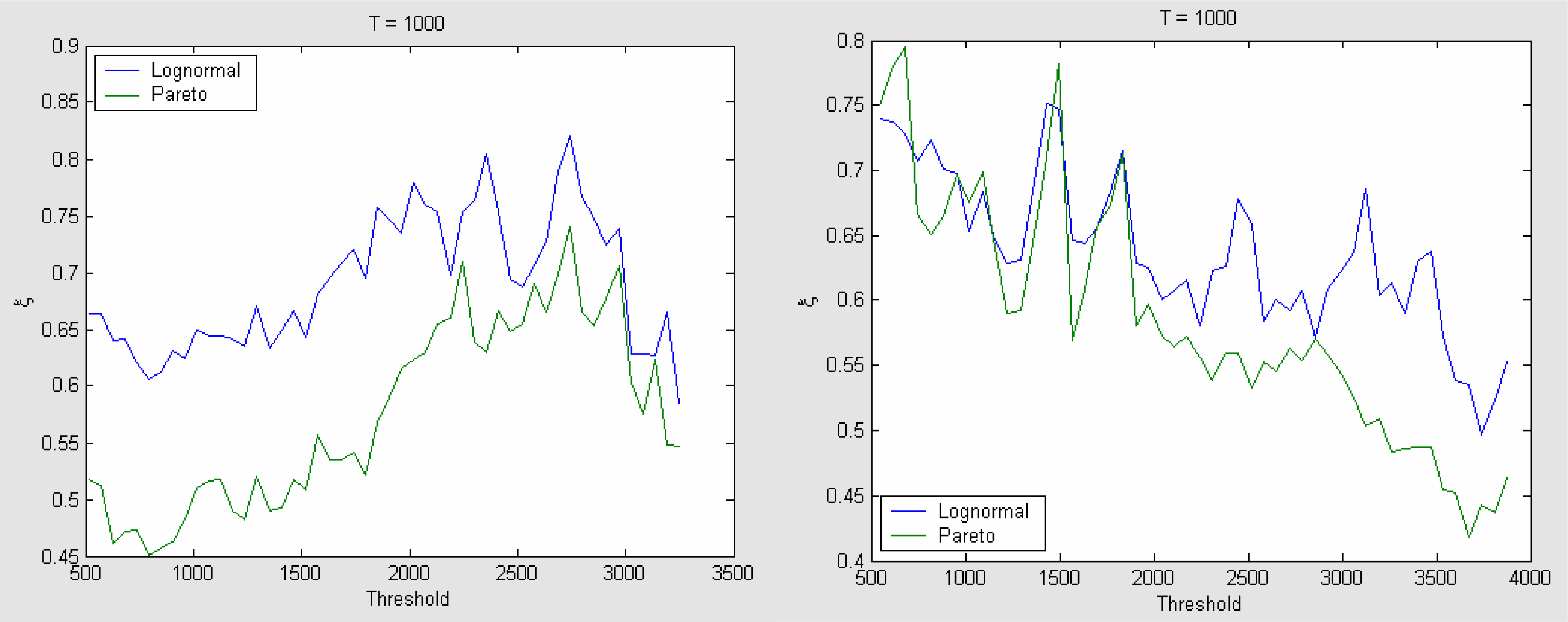
Lognormal data ($T = 10^4$, single simulations)



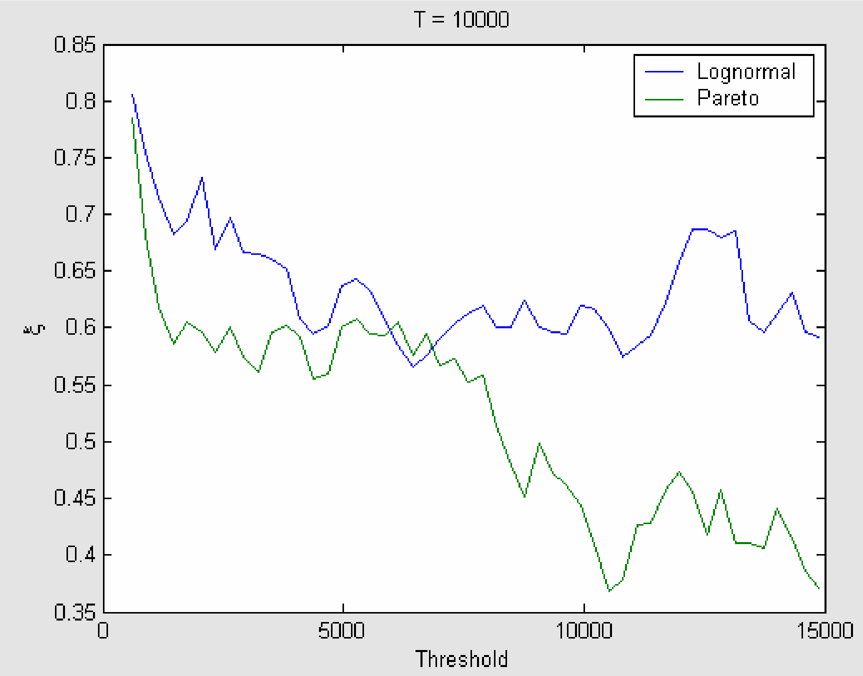
Lognormal data ($T = 10^5$, single simulations)

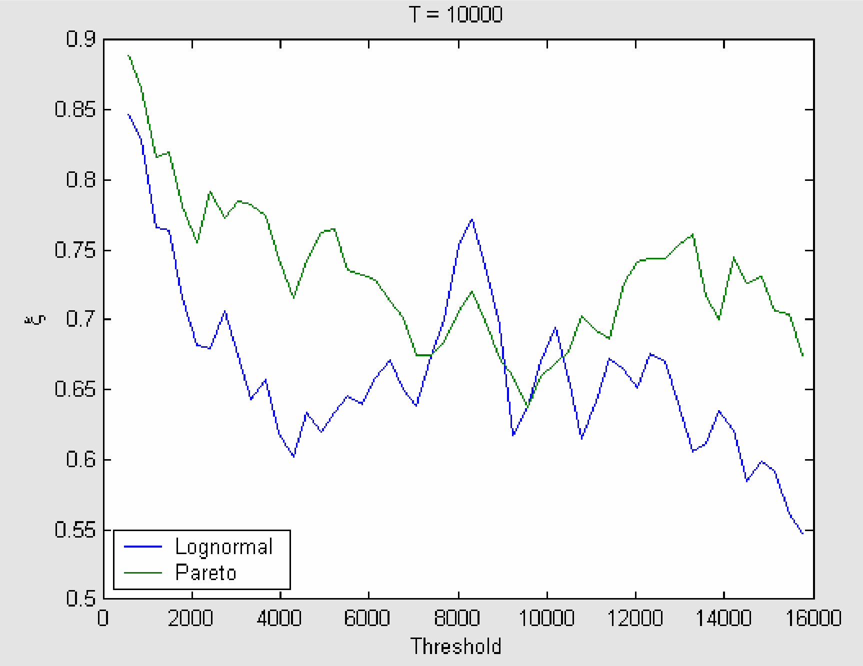


Pareto Fit: Estimate of ξ (T = 10^3)

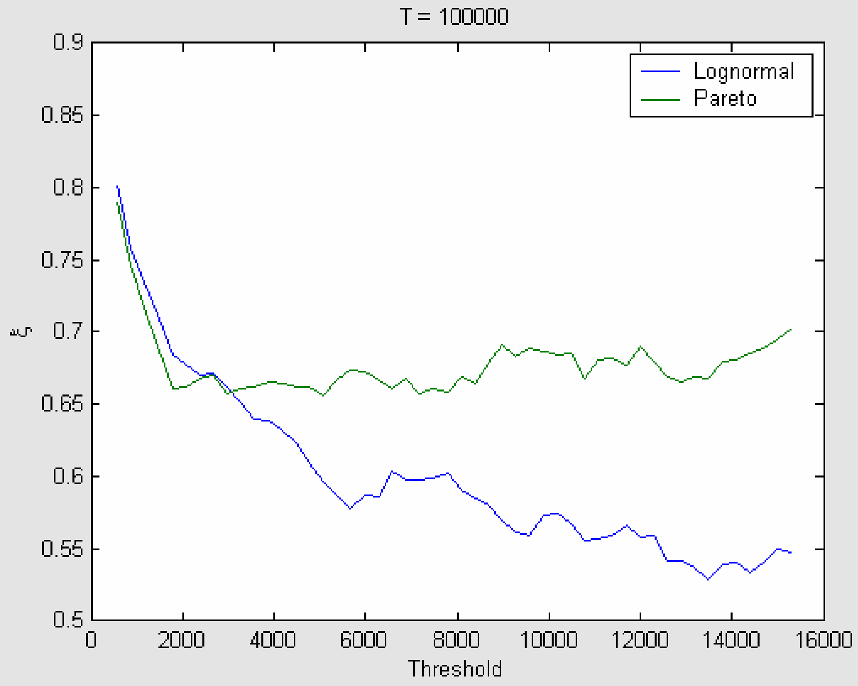


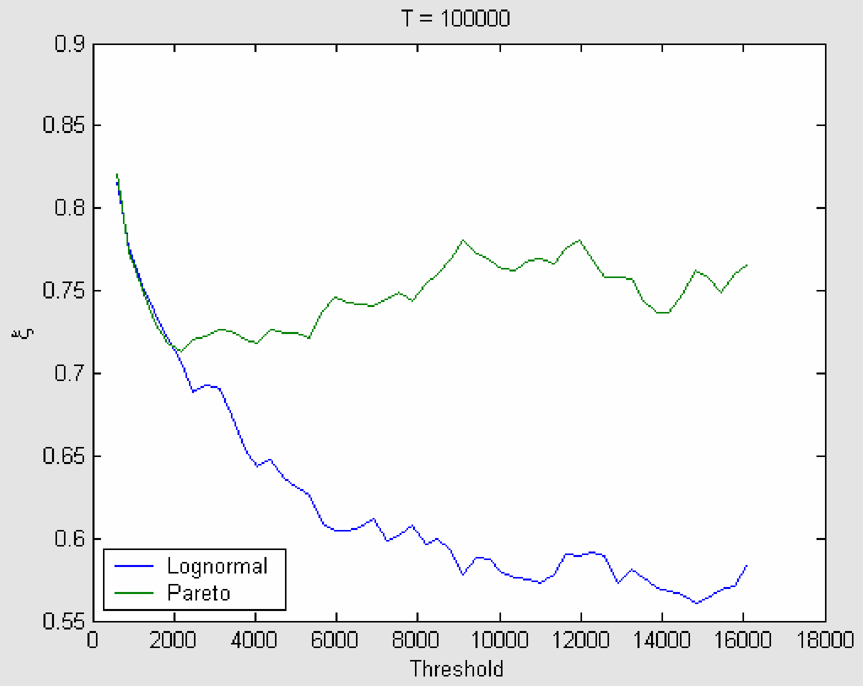
Pareto Fit: Estimate of ξ ($T = 10^4$)





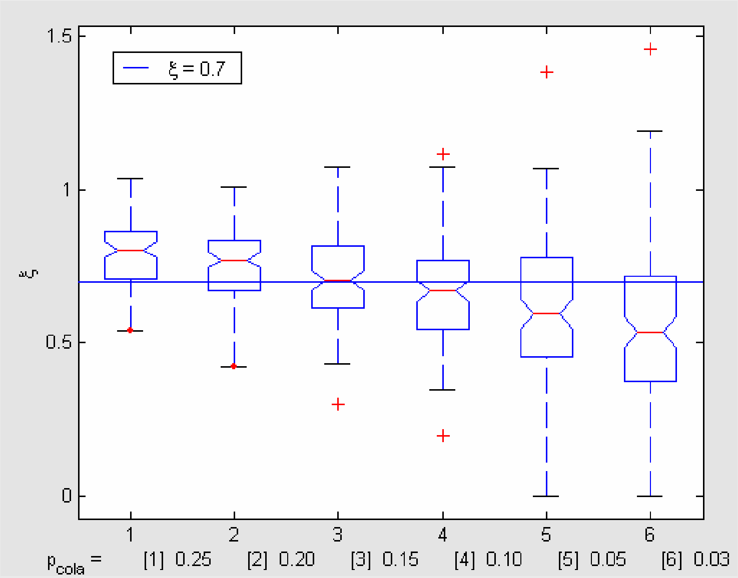
Pareto Fit: Estimate of ξ ($T = 10^4$)



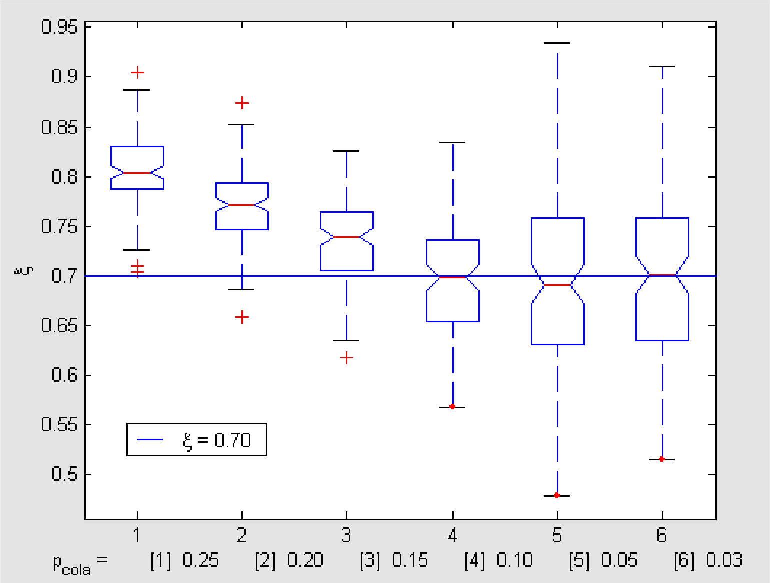


Pareto Fit: Estimate of ξ $(T = 10^3, M = 10^2)$

LOGNORMAL DATA

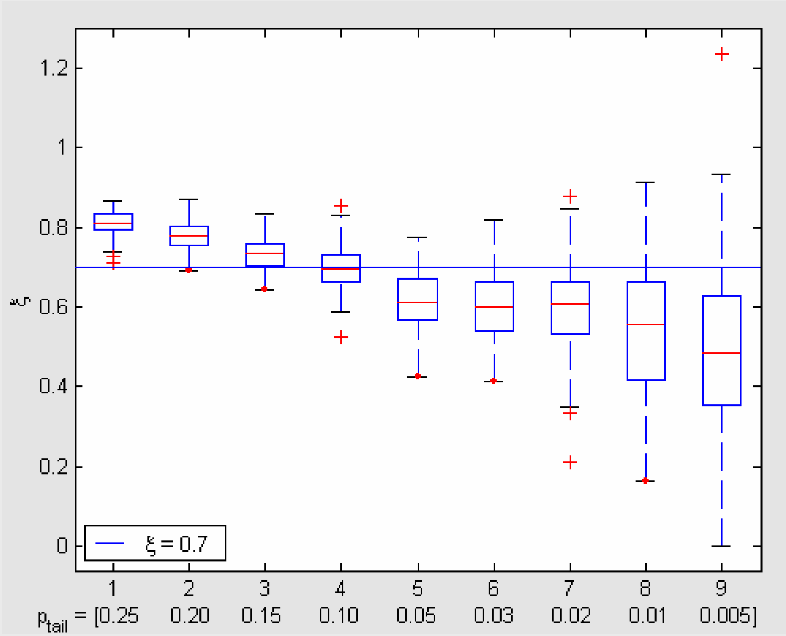


LN + PARETO DATA

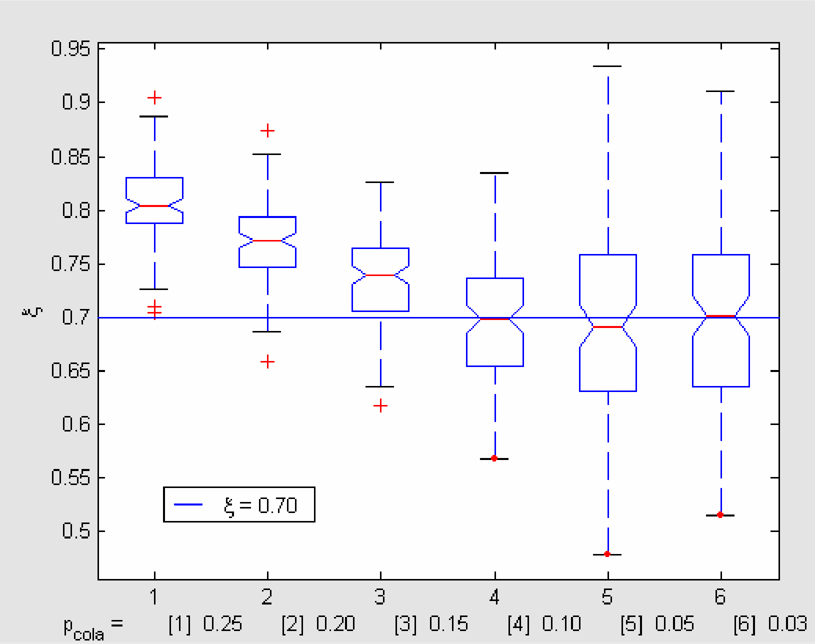


Pareto Fit: Estimate of ξ ($T = 10^4$, $M = 10^2$)

LOGNORMAL DATA

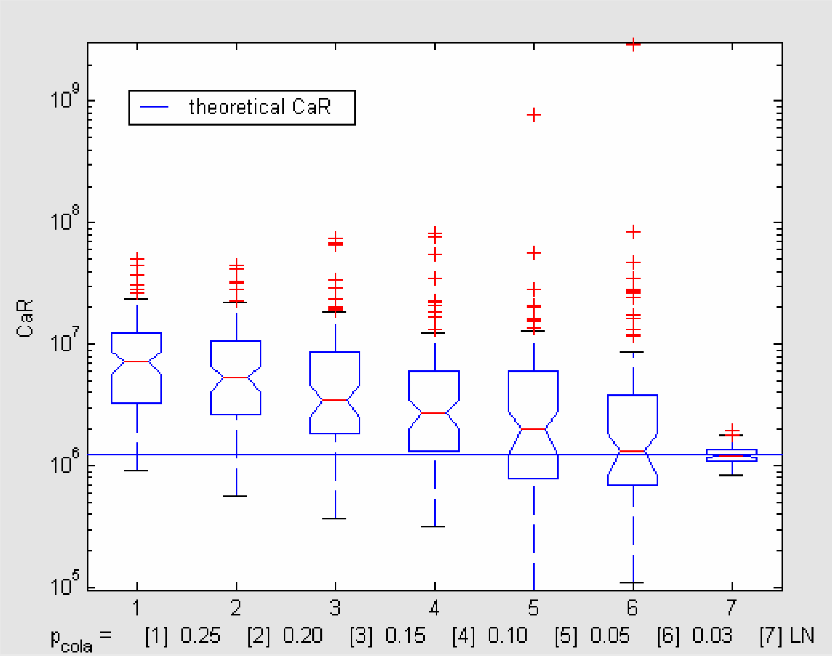


LN + PARETO DATA

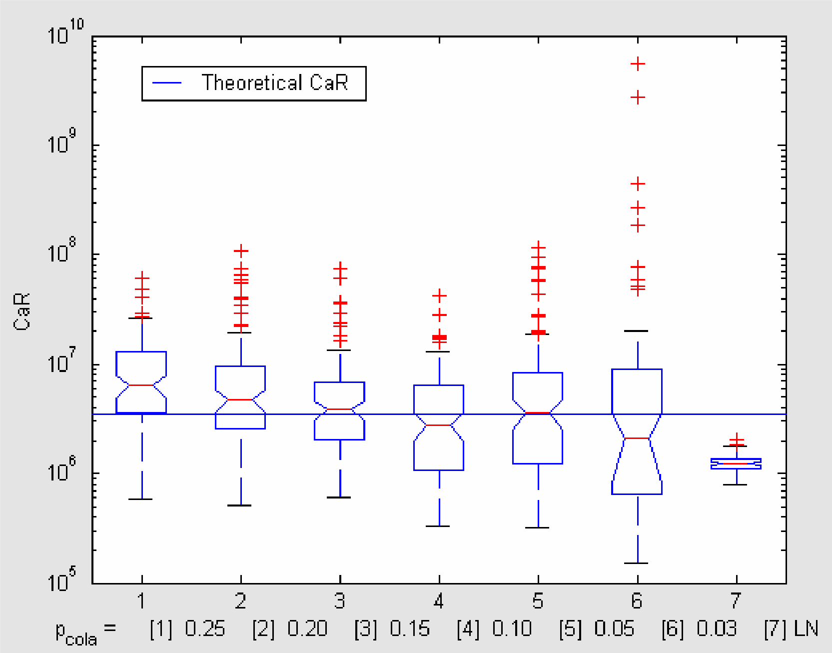


CaR estimates ($\lambda = 200$, $T = 10^3$, $M = 10^2$)

LOGNORMAL DATA

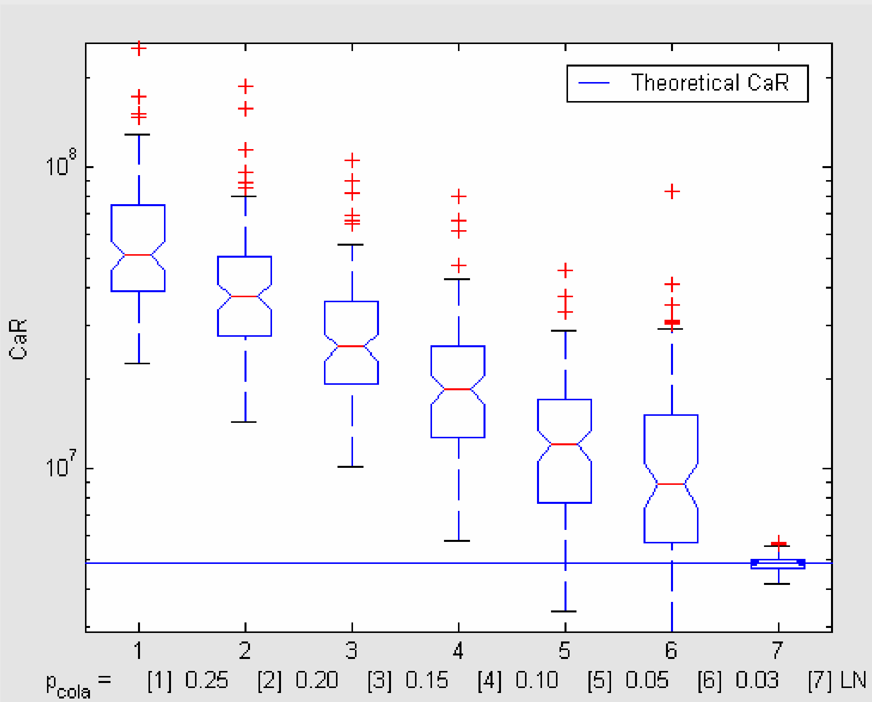


LN + PARETO DATA

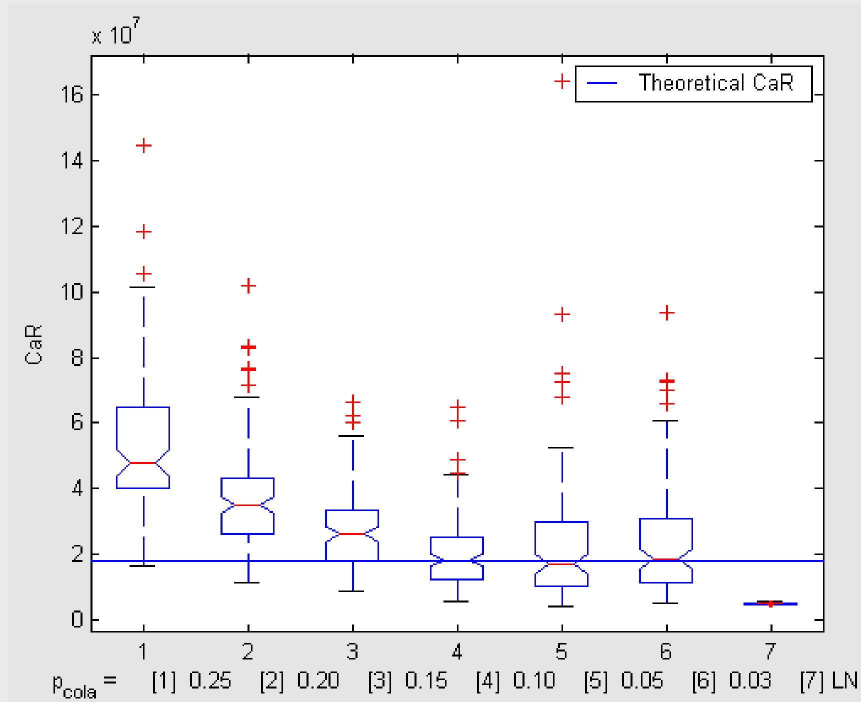


CaR estimates ($\lambda = 2000$, $T = 10^4$, $M = 10^2$)

LOGNORMAL DATA

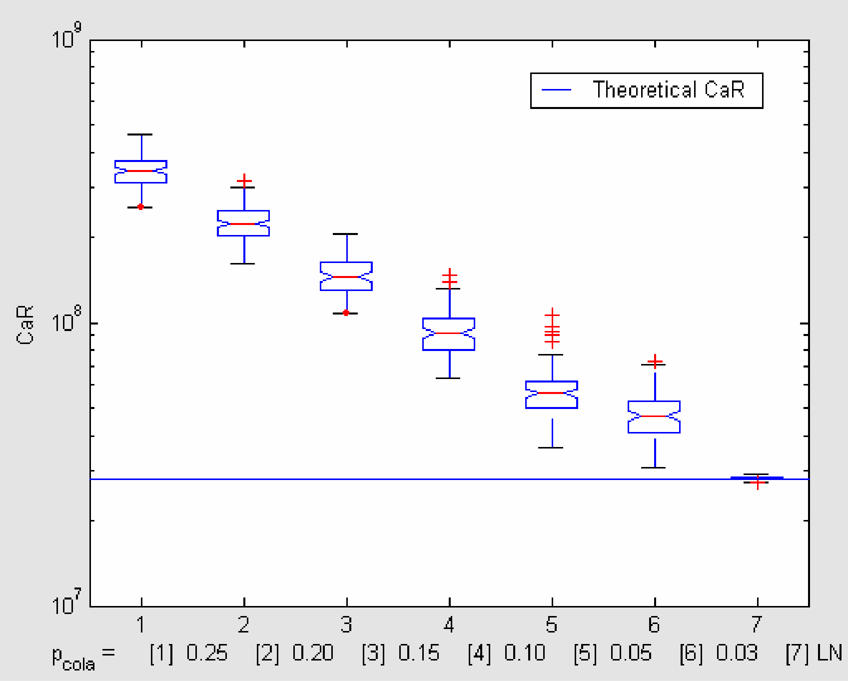


LN + PARETO DATA

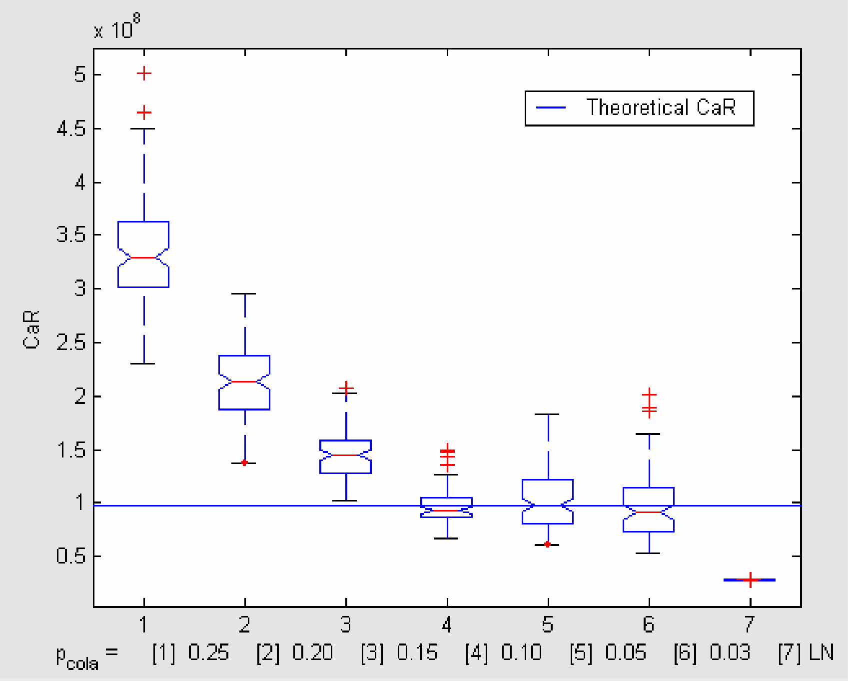


CaR estimates ($\lambda = 20000$, $T = 10^5$, $M = 10^2$)

LOGNORMAL DATA



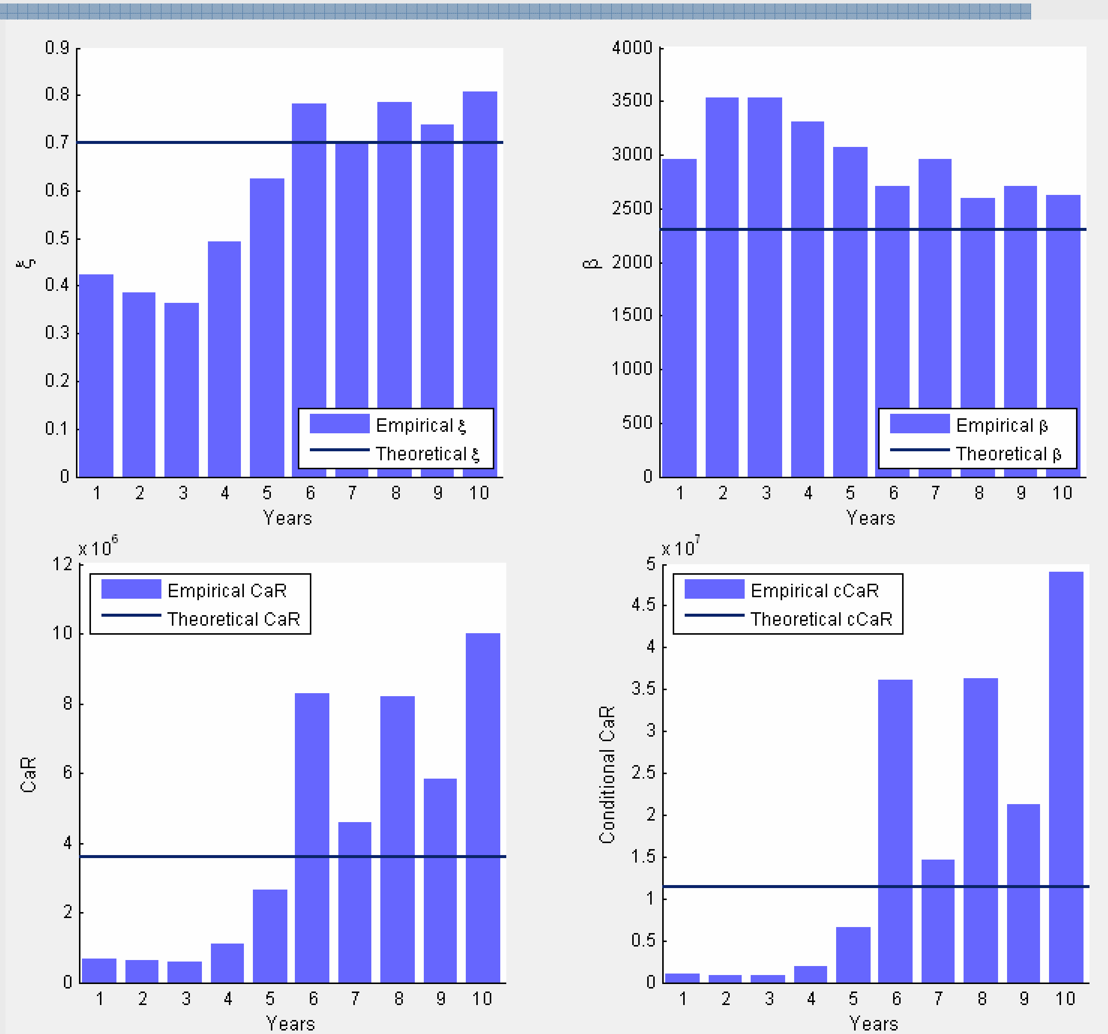
LN + PARETO DATA



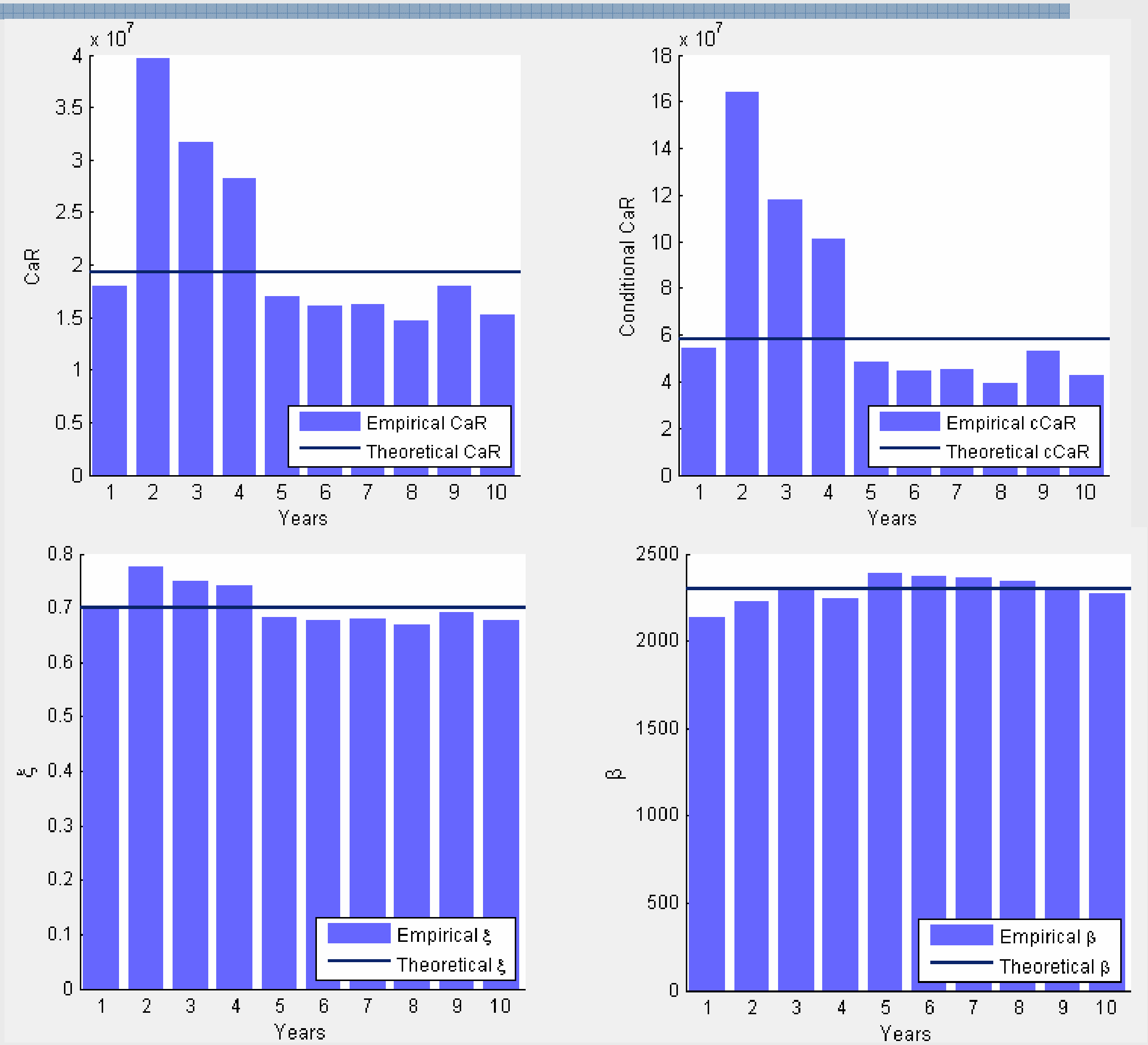
Experiments with Pareto simulated data

- **1,2,…,10 yeas of data** of losses in operational risk events
- Data sampled from a know distribution:
  - **lognormal body ($\mu = 5, \sigma = 2$) +**
    Pareto tail ($u = 1930, \beta = 2300, \xi = 0.7$, 10 % of probability in the tail)
- **Increasing sample size T.**
- Model:
  - Use **Poisson** to model the frequency distribution
    $\lambda = 200, 2000, 20000$
  - Use **empirical body + Pareto tail** (10%) to model the severity distribution.

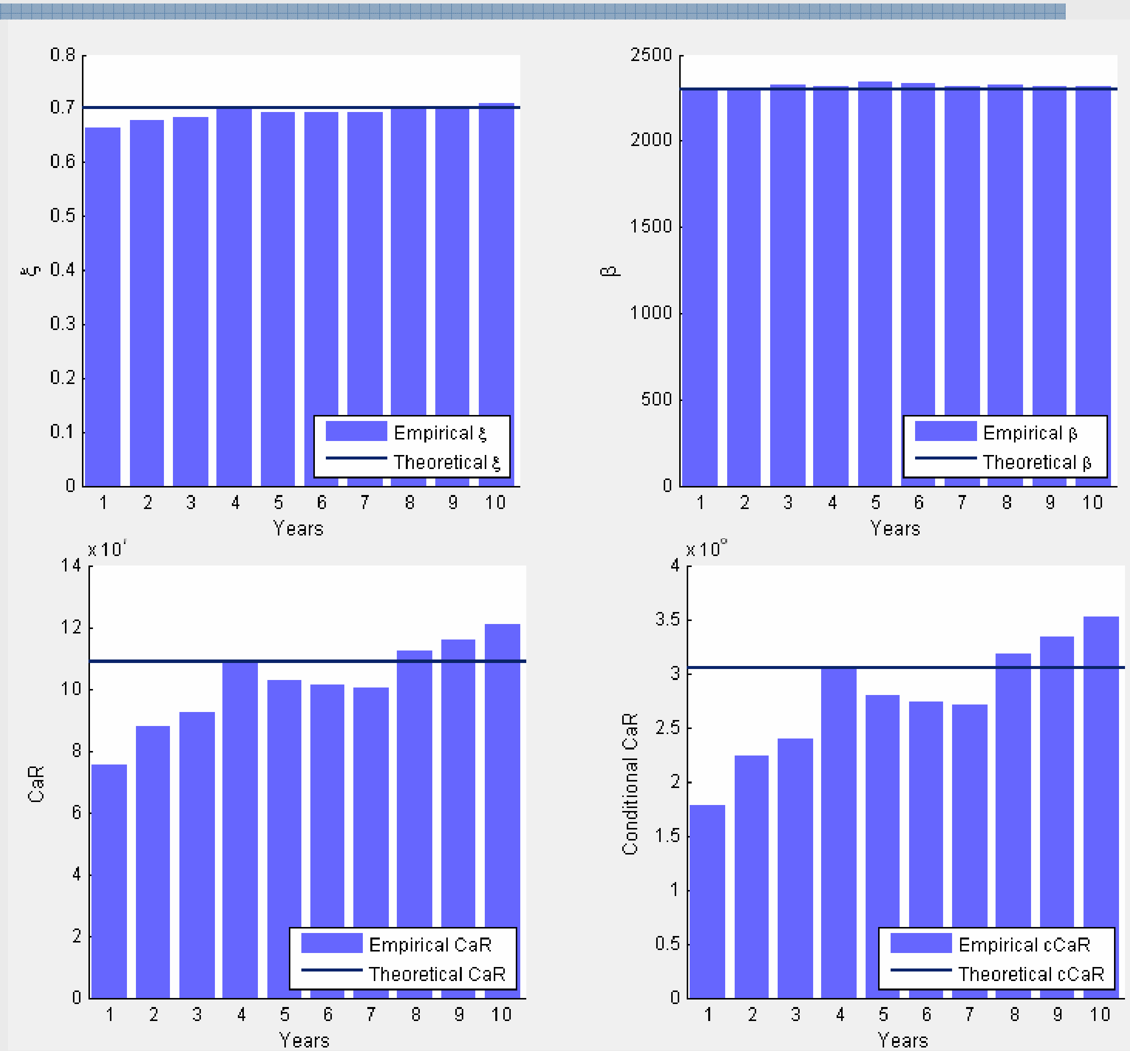
Pareto parameters + CaR estimates ($\lambda = 200$)



Pareto parameters + CaR estimates (λ=2000)



Pareto parameters+ CaR estimates (λ=20000)



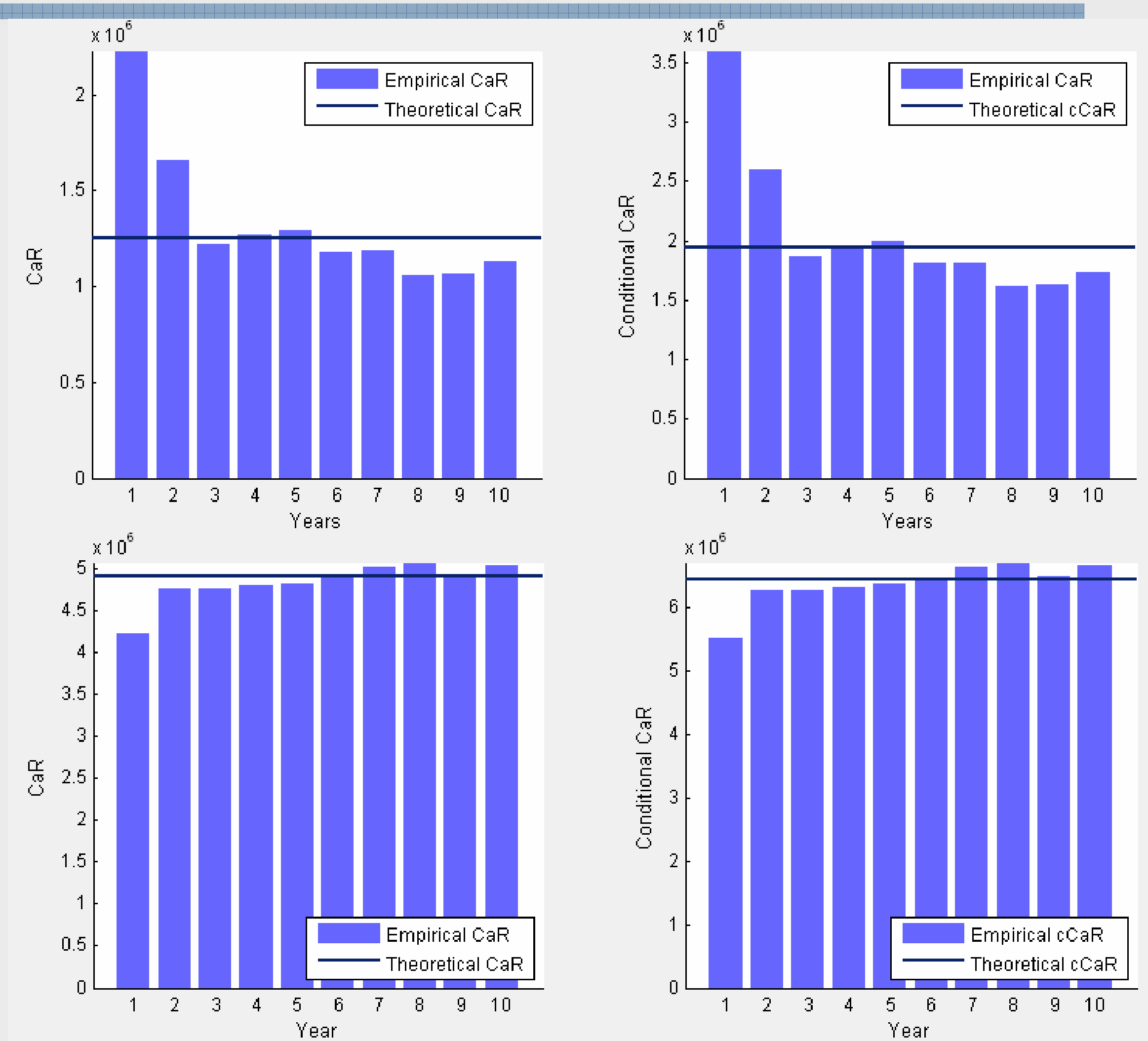
Experiments with Pareto simulated data

- **1,2,…,10 yeas of data** of losses in operational risk events
- Data sampled from a know distribution:
  - **lognormal ($\mu = 5, \sigma = 2$)**
- **Increasing sample size T.**
- Model:
  - Use **Poisson** to model the frequency distribution $\lambda = 200, 2000, 20000$
  - Use **lognormal** to model the severity distribution.

Lognormal parameters+ CaR estimates

λ=200

λ=2000



Sensitivity to single events ($T=10^3$, $M=100$)

	u	β	ξ	CaR ($\times 10^{-3}$)
Theoretical	1930	2300	0.7	3604
Maximum excluded	1900 [1876, 1914]	2352 [2086, 2726]	0.55 [0.45, 0.70]	1144 [661, 3167]
Maximum included	1928 [1913, 1.934]	2303 [2022, 2619]	0.66 [0.55, 0.81]	2492 [1261, 7695]
variation	32 [19, 50]	-65 [-107, -39]	0.1 [0.08, 0.13]	1335 [538, 4537]
% variation	1.67 [0.97, 2.69]	-2.78 [-4.48, -1.72]	17.92 [13.03, 26.96]	104.01 [70.36, 145.72]

Asymptotic approximation (Böcker + Klüppelberg)

- Asymptotic regime : EVT

$$1 - F_{Loss}(x) \approx \lambda(1 - F_X(x)) \qquad x \to \infty, \quad \lambda \text{ small}$$

- Analytic (closed-form) approximation to calculate the percentile of the aggregate loss distribution.

$$CaR(Loss; p) = F_{Loss}^{-1}(p) \approx F_X^{-1}\left(1 - \frac{1-p}{\lambda}\right)$$

Renyi approximation

- Assume frequency follows a negative binomial

$$Loss = \sum_{k=1}^{N} X_k;$$

$$E[X_k] = \mu; \quad P(N = n) = negbinpdf(n; r, p)$$

- Analytic (closed-form) approximation to calculate the percentile of the aggregate loss distribution.

$$CaR(Loss; p) \approx \frac{1}{p} \Gamma_{R,1/\mu}^{-1}(p); \quad p \to 0$$

Lognormal ($\mu = 5, \sigma = 2$) + negative binomial

p	*r*	E[N]	EVT	Renyi	MC (± $error_{MC}$)
0.5	10	10	252	50	255 (± 37)
	100	100	751	293	902 (± 109)
	1000	1000	1996	2414	3144 (± 205)
	10000	10000	4870	22617	16017 (± 607)
0.05	0.5263	10	252	121	292 (± 42)
	5.2632	100	751	334	888 (± 108)
	52.6316	1000	1996	1709	3218 (± 301)
	526.3158	10000	4870	13161	16284 (± 396)
0.005	0.0503	10	252	601	668 (± 60)
	0.5025	100	751	1189	1417 (± 128)
	5.0251	1000	1996	3254	3871 (± 191)
	50.2513	10000	4870	16456	17890 (± 375)

Conclusions

- It is extremely **difficult to distinguish** between **lognormal** and **Pareto** tails for **small data samples**.
- If **data** is actually **lognormal**, but we describe it using a **Pareto model**, **CaR** is typically **overestimated**.
- If **data** is actually **Pareto** , but we describe it using a **lognormal model**, **CaR** is typically **underestimated**.
- **Is EVT directly applicable?**
  - We may not be in the asymptotic regime yet.
  - There is an upper bound for the losses an institution can have (use of distributions with finite support?)

Conclusions

- **Assuming data are Pareto**
  - Empirical estimates of ξ are close to 1 for real operational loss data.
  - Extremely large unrealistic estimates of CaR (economic interpretation?).
  - Very unstable estimates in samples with less than $T = 10^4 - 10^5$ events
    - Choice of threshold.
    - Time evolution.
    - Sensitivity to the presence or absence of extreme events.