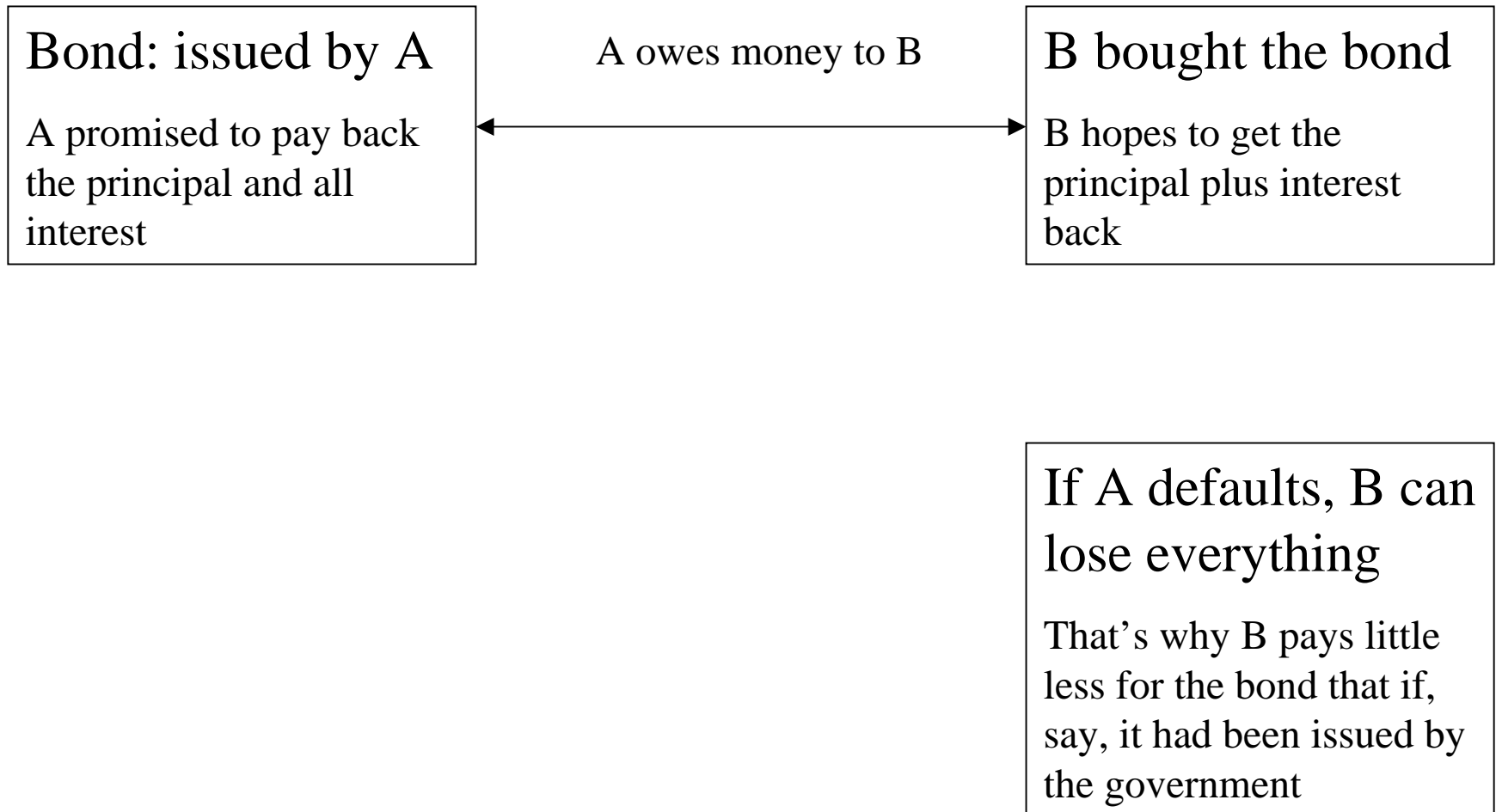


Defaultable forward contracts. Pricing and Modelling.

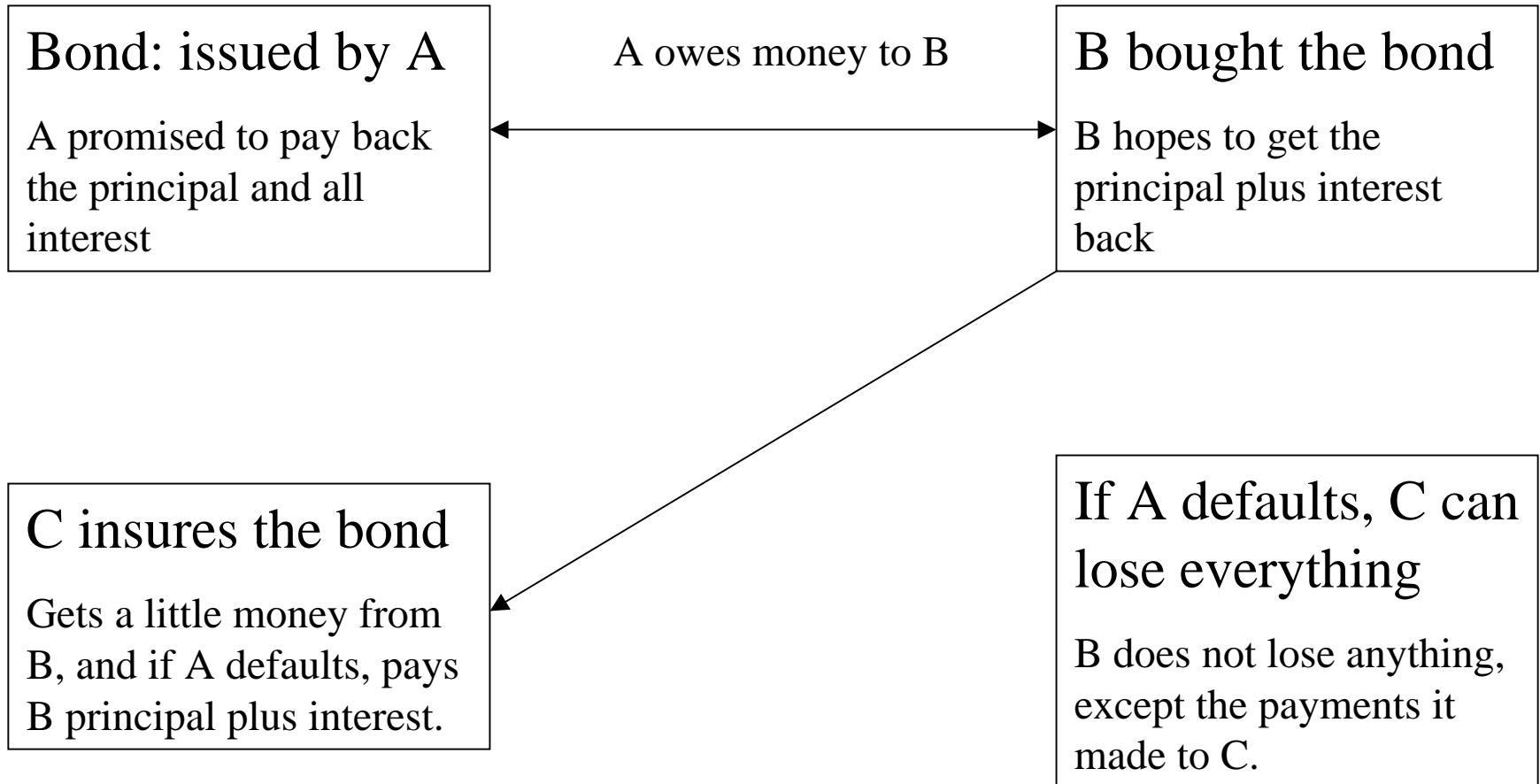
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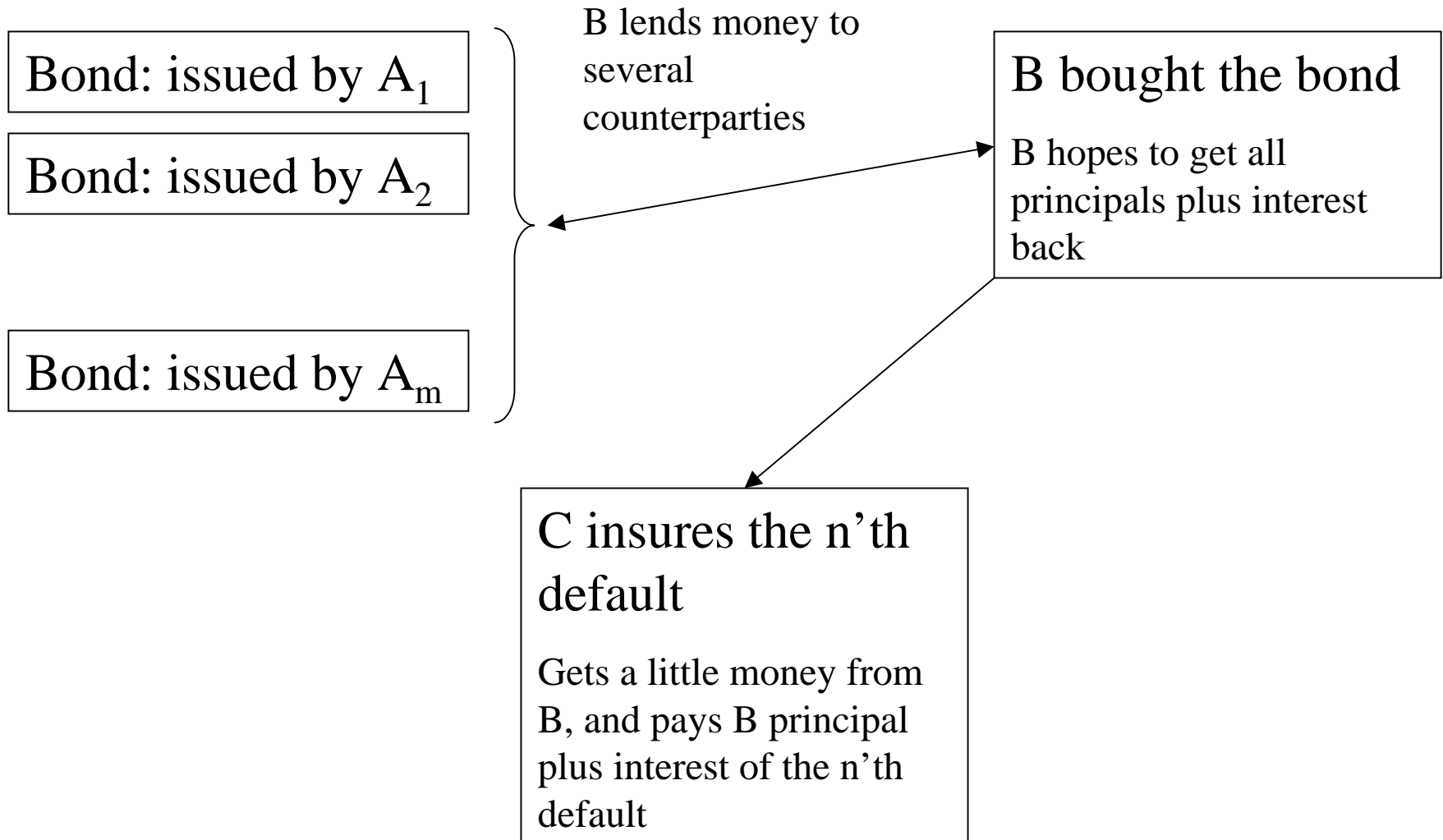
Overview of Credit Markets



Credit protection



n'th to default swap



Introduction

- Future contracts cannot default but forward contracts can; We need to bring default events into the pricing of these OTC transactions.
- Common practice, forward prices in a forward contract are shocked "arbitrarily" to take defaults into account.
- The relation between default events and commodity prices can be represented as a new Term Structure called "defaultable forward prices" (\overline{Fo}).
- The best known example of derivatives in commodities, where default events are considered, are the vulnerable options.

What is a forward contract, FC.

A agrees to buy a designated good on a specified future date, T , at the strike price K , prevailing at the time the contract is initiated, t . No money changes hands initially or during the lifetime of the contract, only at T .

$$FC(t, T) = E_t^Q \left[e^{-\int_t^T r(u) du} (S(T) - K) \right] \quad (1)$$

Definition 1. *What is a forward price, $Fo(t, T)$. It is the value of the strike price such that forward contracts have zero value when they are initiated ($FC(0, T) = 0$).*

What is a Defaultable forward contract, DFC.

Simplest Case: **A** agrees to buy a good at price K at time T from counterparty **B**; if **B** does not default, **A** receives $S(T) - K$ from **B**; in case of a default from **B**, **A** receives nothing.

$$DFC(t, T) = E_t^Q \left[\exp \left\{ - \int_t^T r(s) ds \right\} \cdot \{(S(T) - K) \cdot (1 - N(T))\} \right] \quad (2)$$

Definition 2. *Defaultable forward price, $\overline{Fo}(t, T)$. It is the strike price such that a DFC has zero value when it is initiated ($DFC(0, T) = 0$).*

Other Cases:

If **B** defaults and $K > S(T)$ then **A** pays $K - S(T)$ (real DFC).

Both **A** and **B** may default (two sided DFC).

Examples where this structure is needed: Crash of 1998. Billions of dollars are moved annually on OTC transactions involving defaultable parties.

Mathematical Problem: We wish to price DFC as well as describe the newly implied term structure "defaultable forward prices" under a risk neutral " Q " measure.

Credit Derivatives

Reduced form framework. Notation

- $B(t, T)$, $\bar{B}(t, T)$, bond and defaultable bond prices.
- $S(t)$, Commodity spot price, $Fo(t, T)$ forward price, $F(t, T)$ future prices.
- $N(t)$ - Cox Process with stochastic intensity $\lambda(t)$.
- Forward rate, $f(t, T) = -\frac{\partial}{\partial T} \ln(B(t, T))$.
- Defaultable forward rate, $\bar{f}(t, T) = -\frac{\partial}{\partial T} \ln(\bar{B}(t, T))$.
- Forward convenience yield, $\varepsilon(t, T) = -\frac{\partial}{\partial T} \left(\frac{Fo(t, T)}{S(t)B(t, T)} \right)$.

The processes (drift) for S , f , \bar{f} and $\varepsilon(t, T)$ are known under the Q -measure:

- Heath-Jarrow-Morton 1991 provides the drift of $f(t, T)$ under general conditions Most interest rate models are particular cases of this Framework.
- Schwartz 1997 generalizes HJM for commodities by finding the drift of $\varepsilon(t, T)$. The drift of $S(t)$ is known from Black-Scholes.
- Schonbucher 2001 generalizes HJM, describing the drift of $\bar{f}(t, T)$ in a risk neutral world.

These results will be used to describe the drift of the defaultable forward price $\overline{Fo}(t, T)$ in the risk neutral world.

Inspired by the frameworks of:

HJM 1991 for Bonds $B(t, T) = e^{\int_t^T f(t, s) ds}$

Schwartz 1997 for forward prices $Fo(t, T) = S_t \cdot e^{\int_t^T (f(t, s) - \delta(t, s)) ds}$

Schonbucher for defaultable Bonds $\bar{B}(t, T) = e^{\int_t^T \bar{f}(t, s) ds}$

We propose the following model for $\bar{Fo}(t, T)$. The idea is to decompose the term structure into a suitable set of factors:

$$\bar{Fo}(t, T) = S_t \cdot \exp \left\{ \int_t^T (\bar{f}(t, s) - \bar{\varepsilon}(t, s)) ds \right\}, \quad (3)$$

The term $\bar{\varepsilon}(t, s)$ is called defaultable instantaneous convenience yields.

Theorem 1. *The drift of $\overline{F}_0(t, T)$, $\overline{\mu}_{F_0}^Q$ and the drift of $\overline{\varepsilon}(t, T)$, $\overline{\mu}_{\varepsilon}^Q$, in the absence of arbitrage, are:*

$$\overline{\mu}_{F_0}^Q(t, T) = F(t, T) \times \frac{\partial G_0(t, T, \sigma_S, \sigma_\lambda, \sigma_\varepsilon, \mu_\varepsilon^Q, \mu_f^Q, \overline{\mu}_f^Q)}{\partial t} \quad (4)$$

$$\begin{aligned} \overline{\mu}_{\varepsilon}^Q(t, T) &= -\mu_\varepsilon^Q(t, T) + \mu_f^Q(t, T) - \overline{\mu}_f^Q(t, T) \\ &+ \frac{\partial^2 \left\{ G_1(t, T, \sigma_S, \sigma_\lambda, \sigma_\varepsilon, \mu_\varepsilon^Q, \mu_f^Q, \overline{\mu}_f^Q) \right\}}{\partial T \partial t} \end{aligned} \quad (5)$$

respectively.

Other Credit Derivatives. This previous concept can be used as the backbone of a new breed of derivatives, for example:

1. **Options on DFC:** An standard option with maturity t on a defaultable forward contract starting at t maturity T , K stand by the strike price.

$$E_0^Q \left[\exp \left\{ - \int_0^t r(s) ds \right\} \cdot (\overline{Fo}(t, T) - K)^+ \right]$$

2. **Vulnerable options on DFC:** The issuer of the options may default before option's maturity day. There are two sources of default, the option seller, N_1 , and the DFC underlying.

$$E_0^Q \left[\exp \left\{ - \int_0^t r(s) ds \right\} \cdot (\overline{Fo}(t, T) - K)^+ \cdot (1 - N_1(T)) \right]$$

Structural Framework

Merton approach. Basic assumptions.

1. Defaults occurs at the maturity of the option, T , only if $V(T) < D$, where $V(t)$ stands for the option writer's assets (Merton approach 1974).
2. Zero recovery rate.

The Payoff of a defaultable forward contract in this context is:

$$E_t^Q \left[e^{\left\{ -\int_t^T r(s) ds \right\}} \cdot (S(T) - K) \cdot (1 - 1_{V(T) < D}) \right] \quad (6)$$

Results. We price one and two-sided DFC as well as vulnerable options on spot and futures prices.

Conclusions

- The notion of defaults, inherent on forward contracts, was added leading to an alternative derivative called defaultable forward contract.
- Defaultable forward prices were defined and modelled following standard frameworks.
- The idea was extended to other families of defaultable contracts as two sided DFC and real DFC.