Compositional Description, Valuation, and Management of Financial Contracts: The MLFi Language

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Contents

- The Challenge
- Compositional Description of Financial Contracts
- Valuation
- Management
- Other Applications
- Summary

The Challenge

What We Hear and See

"Self-explaining data"

"B2B automation"

"Multiple, new products"

"Link 3rd party and proprietary systems"

"Disparate users"

"Operational risk"

"Flexibility… but control"

"Standards"

"Reusability"

"Aggregation"

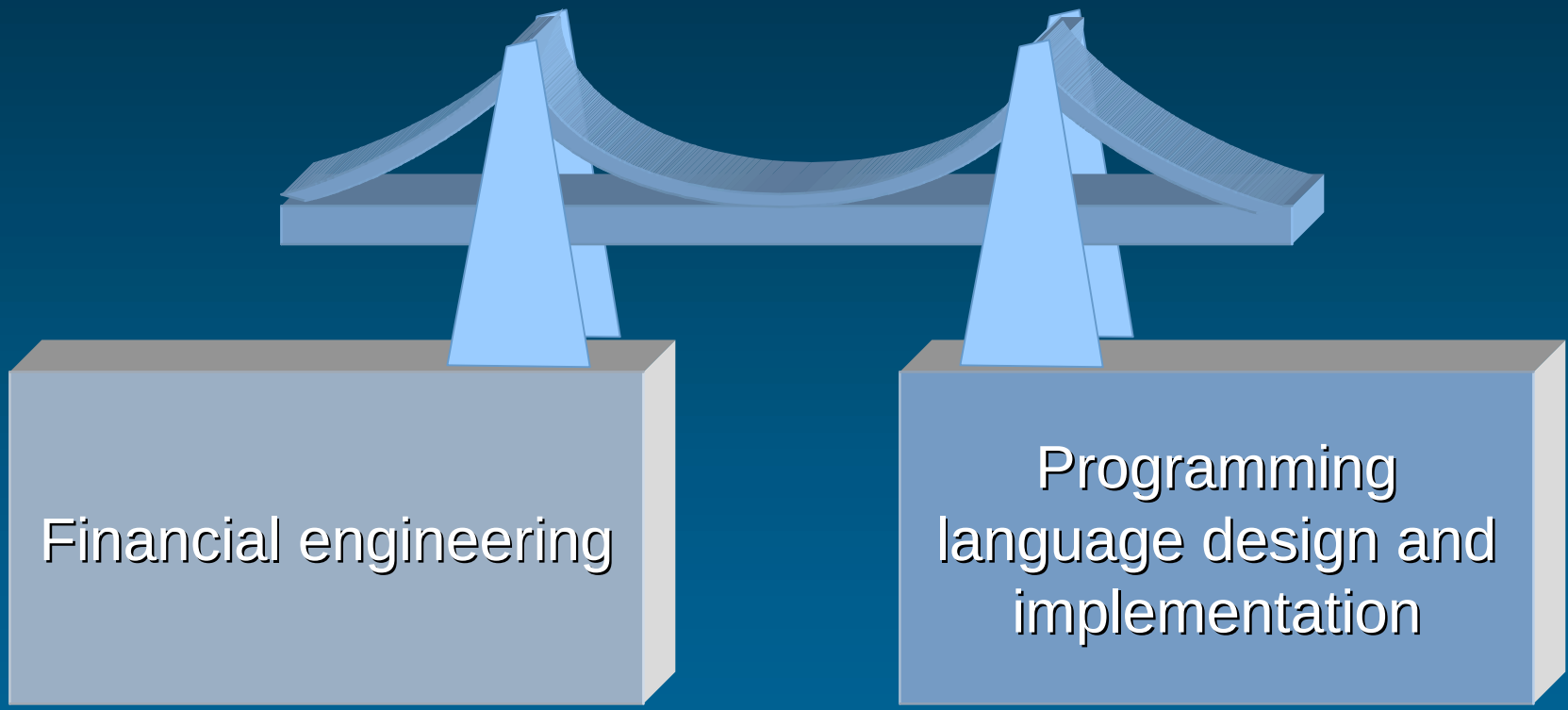
"Extensible"

"Formal OTC exchanges"

Business Drivers

- Growing number and complexity of products
  - that require increasingly precise descriptions
- New and evolving processes
- Mastering operational risk as a source of competitive advantage for any financial institution
- Unified approach of insurance, finance and "retail" contracts
- Regulation
- Importance of cross-functional processes (VaR, regulatory reporting, Raroc, credit risk)
- B2B “automation”

The Big Picture



Compositional Description of Financial Contracts

Financial Contracts Are Complex

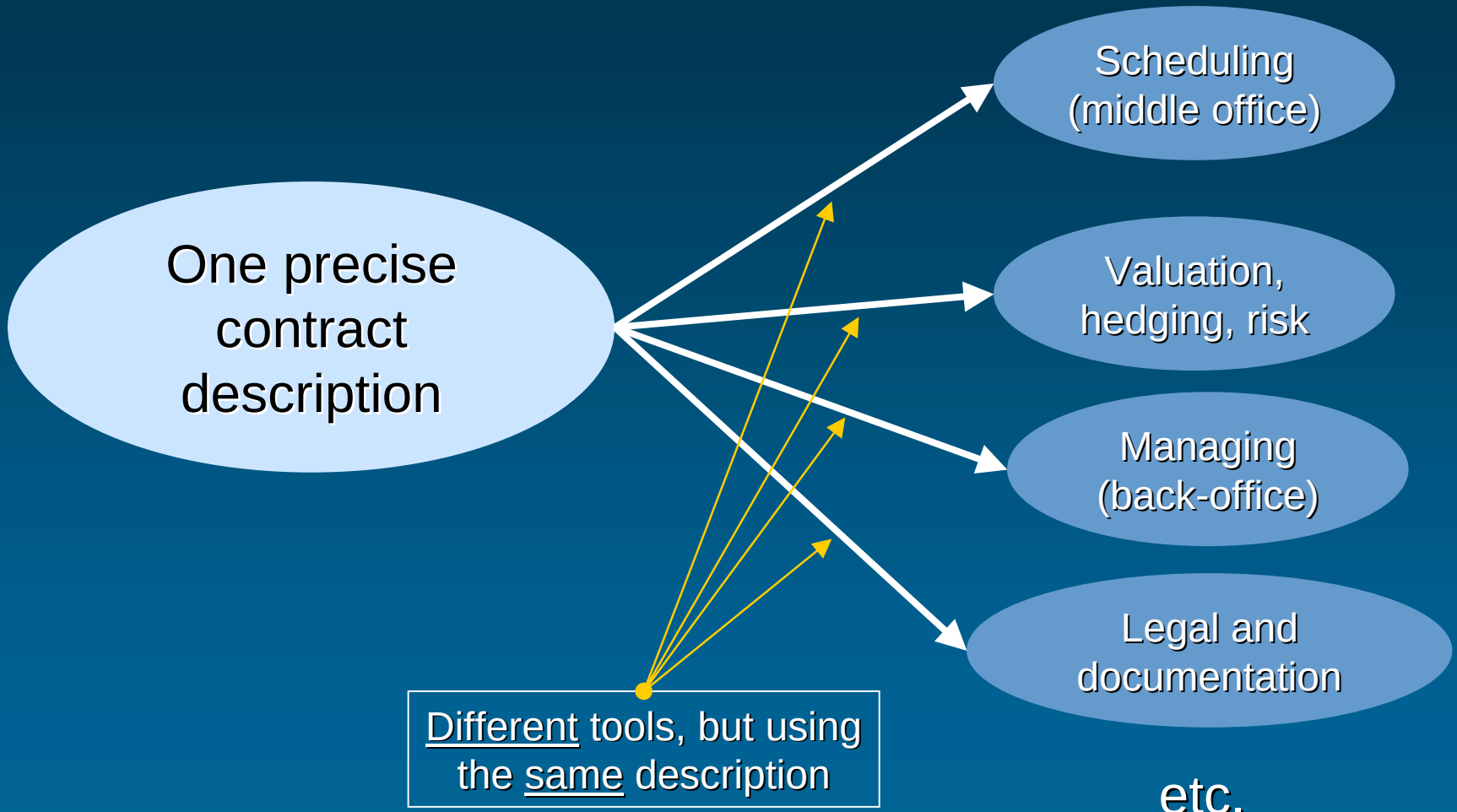
Example:

- An option, exercisable any time between t1 and t2
  - on an underlying consisting of a sequence of fixed payments
  - plus some rule about what happens if you exercise the option between payments
- plus a fixed payment at time t3

- Complex structure
- Subtle distinctions
- Need for precision

…for many uses!

What We Want to Do

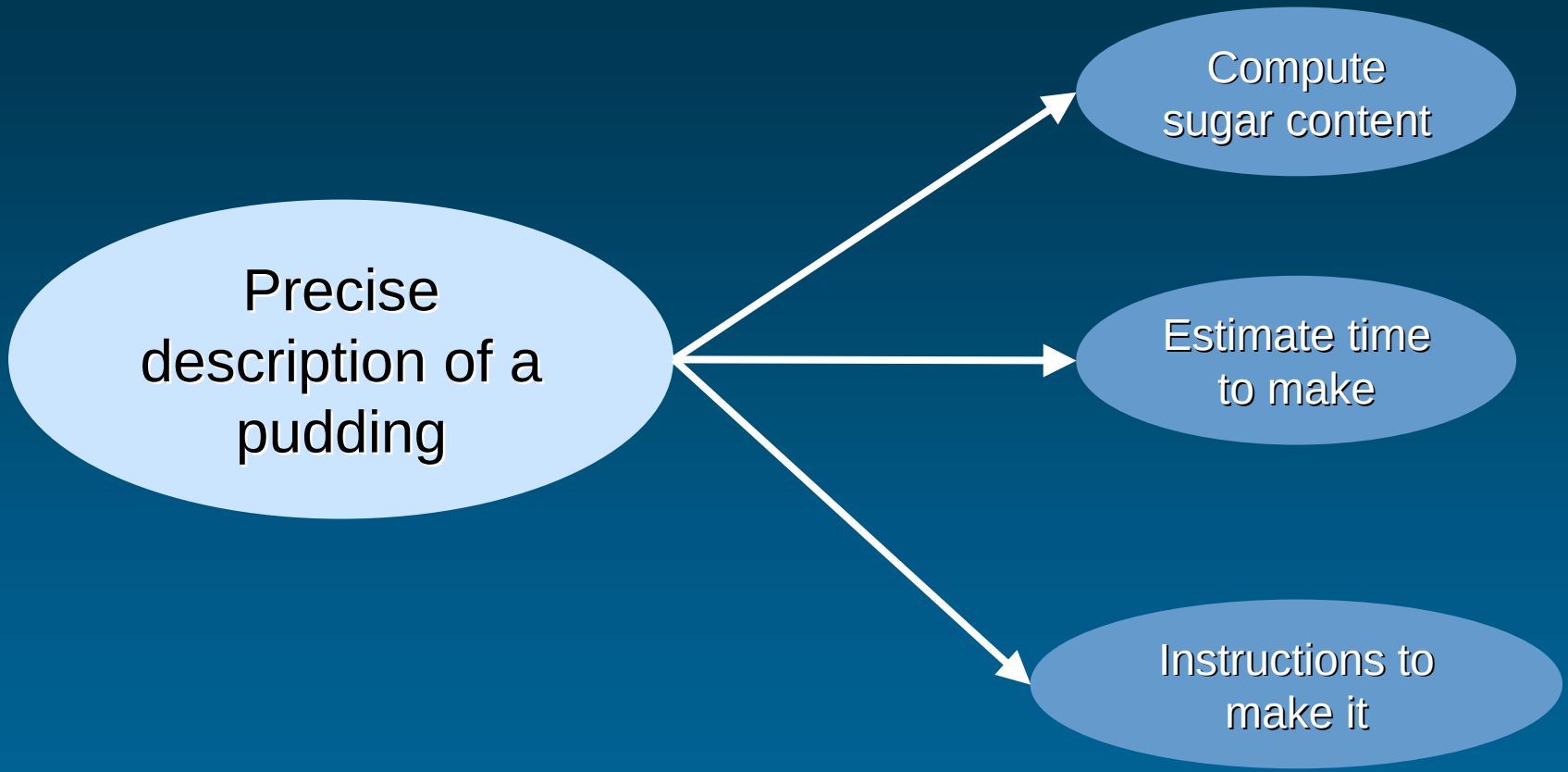


What We Want to Do (2)

Precise description means:

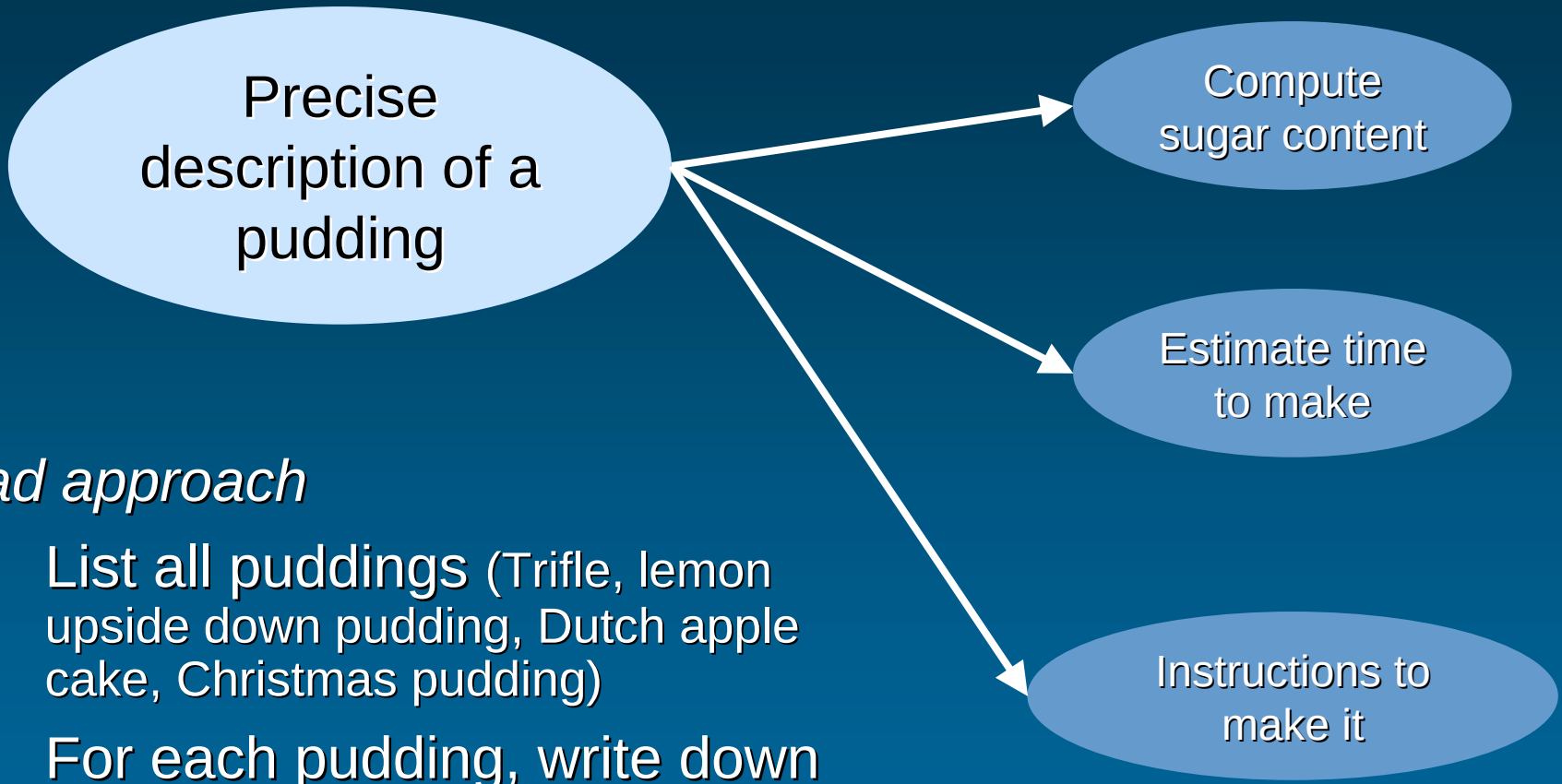
- A syntax (easy), may be a “language”, a graphical notation, etc. that combines some elementary "atoms" with "operators"
- A clear semantics (difficult): what do the operators *mean*?
- But one unambiguous description shared between different people and different applications

An Analogy



etc.

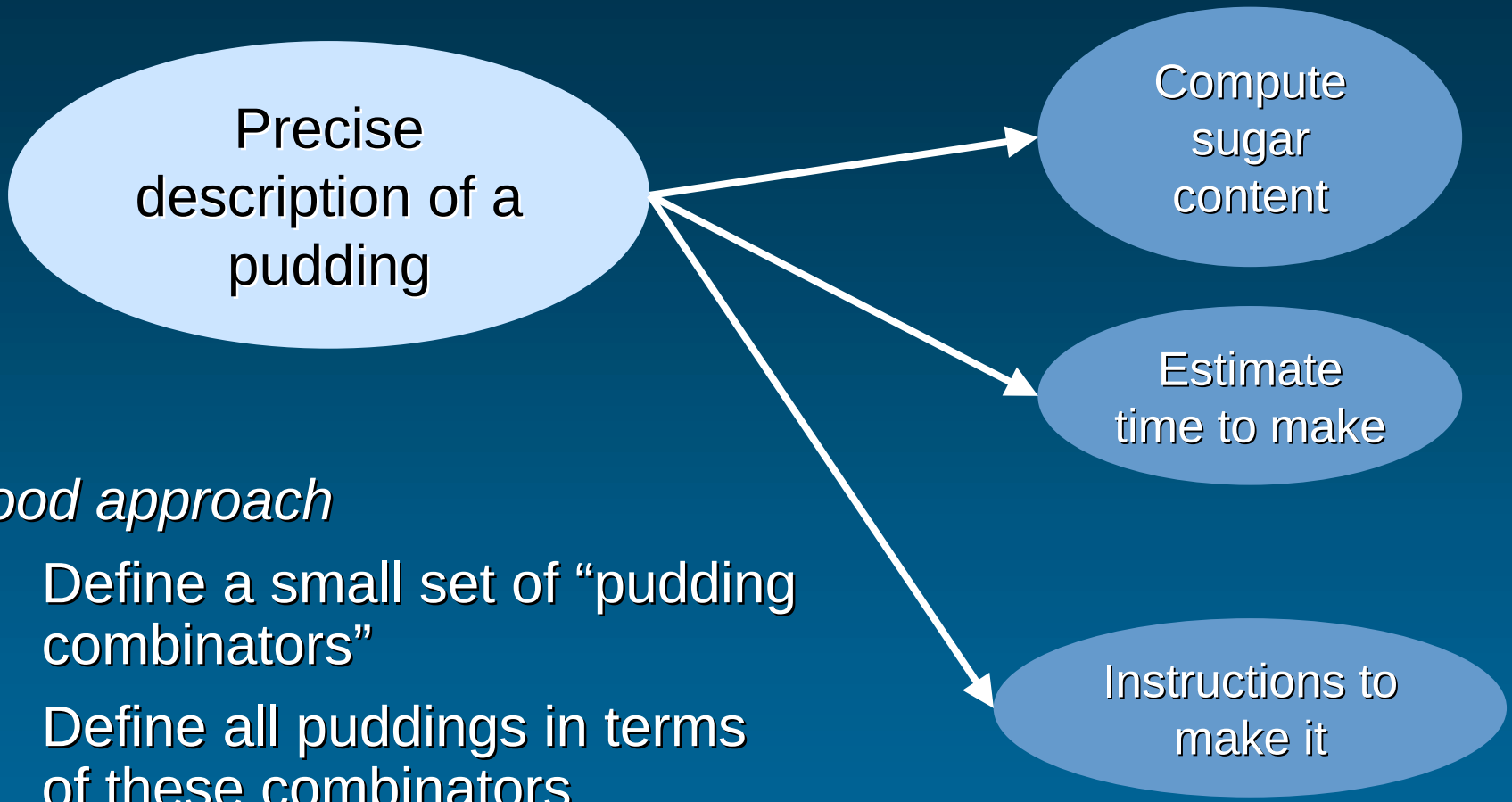
An Analogy (2)



Bad approach

- List all puddings (Trifle, lemon upside down pudding, Dutch apple cake, Christmas pudding)
- For each pudding, write down sugar content, time to make, instructions, etc

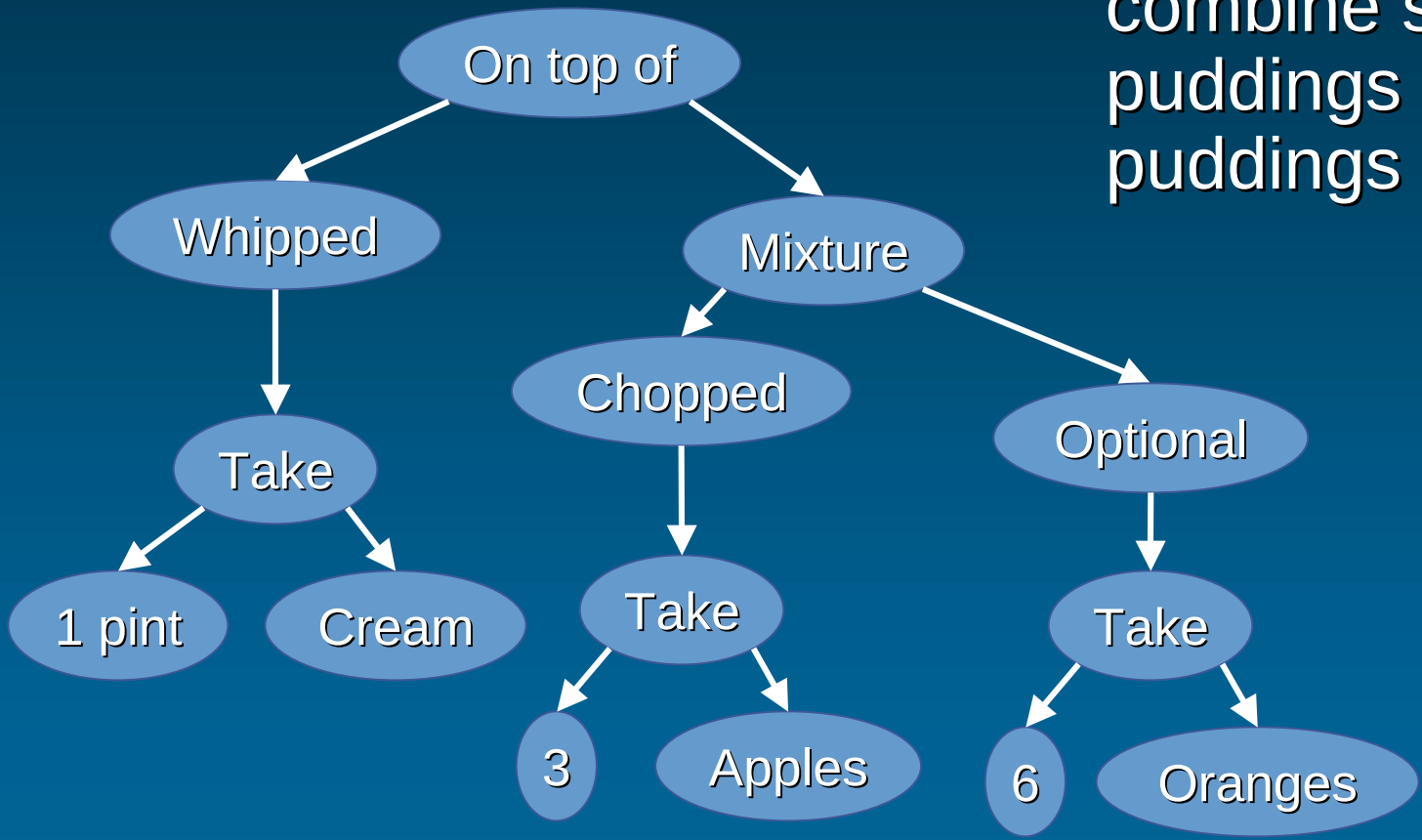
An Analogy (3)



Good approach

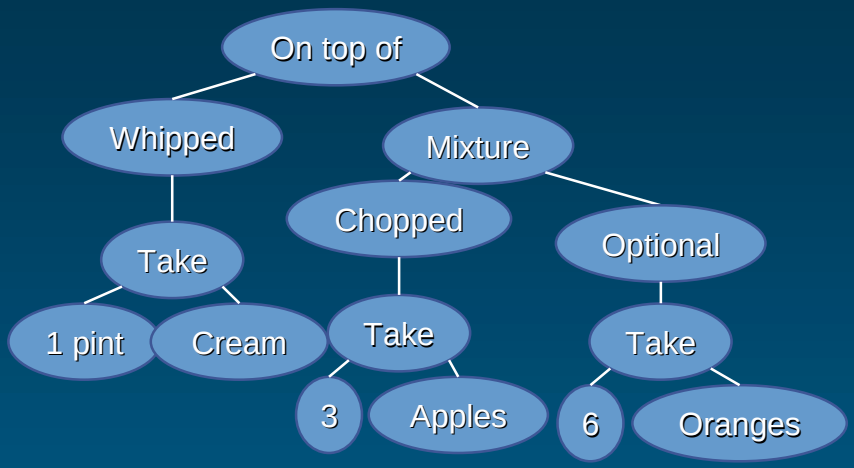
- Define a small set of “pudding combinators”
- Define all puddings in terms of these combinators
- Calculate sugar content from these combinators too

Creamy Fruit Salad



Combinators combine small puddings into bigger puddings

Trees Can Be Written As Text



Notation:

- `parent child1 child2`
- `function arg1 arg2`

```
salad       = onTopOf topping main_part
topping     = whipped (take pint cream)
main_part   = mixture apple_part orange_part
apple_part  = chopped (take 3 apple)
orange_part = optional (take 6 oranges)
```

Slogan: a domain-specific language for describing puddings

Processing Puddings

- Wanted: S(P), the sugar content of pudding P

S(**`onTopOf p1 p2`**)	= S(**`p1`**) + S(**`p2`**)
S(**`whipped p`**)	= S(**`p`**)
S(take q i)	= q * S(i)
Etc.	

- When we define a new recipe, we can calculate its sugar content with no further work
- Only if we add new combinators or new ingredients would we need to enhance S

Processing Puddings (2)

- Wanted: S(P), the sugar content of pudding P

```
S(onTopOf p1 p2)   = S(p1) + S(p2)
S(whipped p)       = S(p)
S(take q i)        = q * S(i)
Etc.
```

S is *compositional*

To compute S for a compound pudding,

- Compute S for the sub-puddings
- Combine the results in some combinator-dependent way

Doing The Same for Contracts

The big question

What are the appropriate primitive combinators for financial contracts?

- How to achieve a compositional approach?
- How to master the different uses in one description?
- Build a theory of financial contract description

Building a Simple Contract

```
c1 : contract
c1 = zcb(2010-01-25, 100, GBP)
```

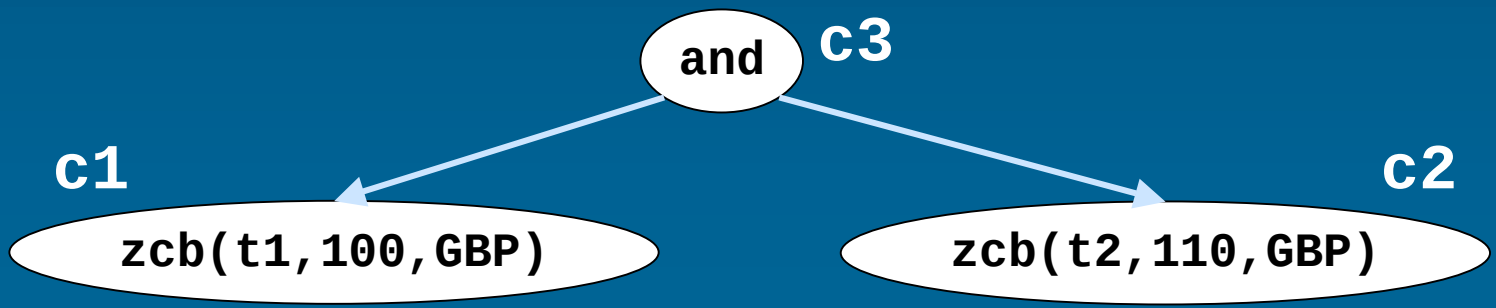
```
zcb : (date * float * currency) -> contract
(* Zero coupon bond *)
```

- `contract` is a build-in type of our language (like `integer` in a classical programming language) !
- `date` is also a build-in type (and `2010-01-25` is recognized as a constant of this type, like `78.5` is recognized as a `float`).

Building a Simple Contract (2)

```
c1,c2,c3 : contract
c1 = zcb(2010-01-25, 100, GBP)
c2 = zcb(2011-01-24, 110, GBP)

c3 = and(c1, c2)
```

```
and : (contract * contract) -> contract
(* Both c1 and c2 *)
```



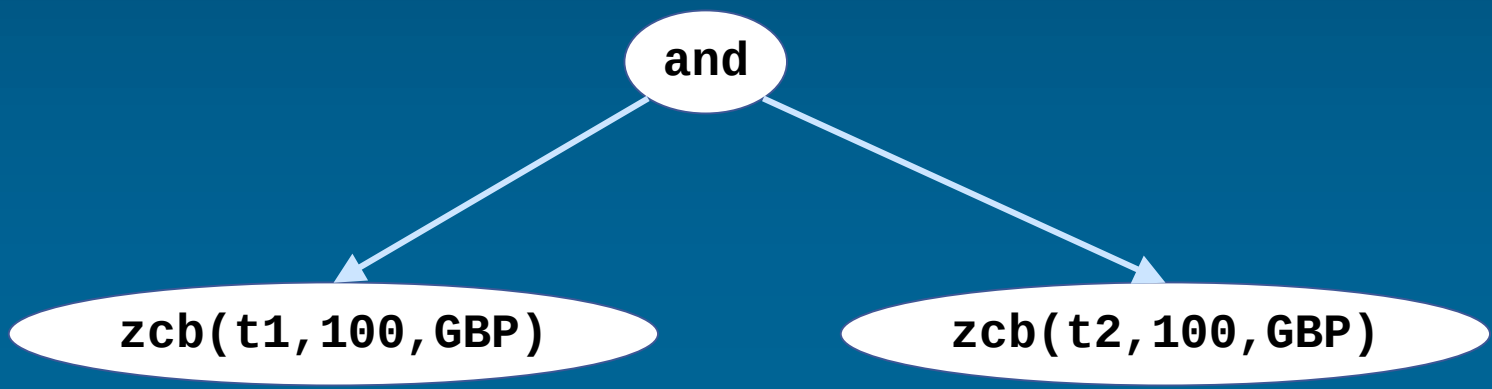
Building a Simple Contract (3)

```
c3 = and(c1, c2)
```

can be written

```
c3 = c1 'and' c2
```

Notational convenience: write combinators with two arguments in an infix position (like `'and'`)

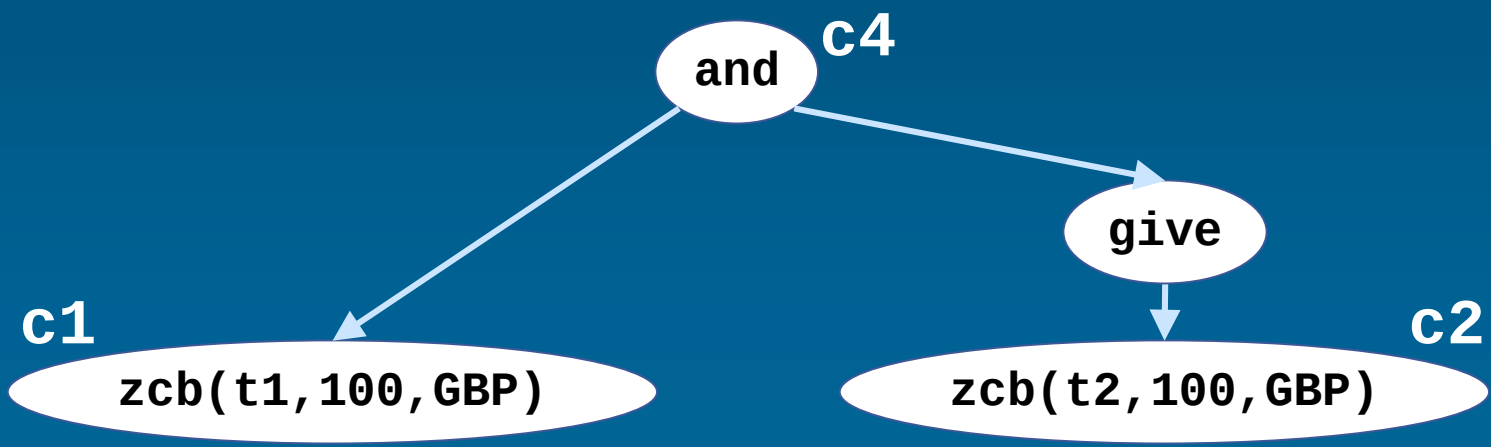


Inverting a Contract

```
c4 = c1 'and' (give c2)
```

```
give : contract -> contract
(* Invert role of parties *)
```

- `and` is “like” addition
- `give` is “like” negation



New Combinators from Old

```
andGive : (contract * contract) -> contract
andGive(u1, u2) = u1 'and' (give u2)
```

- `andGive` is a new combinator, defined in terms of simpler combinators
- To the “user” it is not different from a primitive, built-in combinator
- This is the key to extensibility: users can write their own libraries of combinators to extend the built-in ones

Choice

An option gives the holder the flexibility to

- Choose which contract to acquire (or, as a special case, whether to acquire a contract)
- Choose when to acquire a contract (exercising the option = acquiring the underlying)

Choose Which

```
or : (contract * contract) -> contract
(* Either c1 or c2 *)

zero : contract
(* A contract without any right or
   obligation *)
```

- First attempt at a European option

```
european : contract -> contract
european u = u 'or' zero
```

- But we need to specify <u>when</u> the choice may be exercised

Temporal Acquisition of Contracts

```
acquire : (region * contract) -> contract
(* Obligation, the first time the region is
hit, to acquire the argument contract. *)
```

A region can be complex. But here we consider a very simple one: `{[t]}` is the region "date t"

```
european : (date * contract) -> contract
european(t, u) = acquire({[t]}, u 'or' zero)
```

- A contract is *acquired* on its *acquisition* date
- If you acquire the contract (c1 'or' c2) you must *immediately* choose which of c1 or c2 to acquire

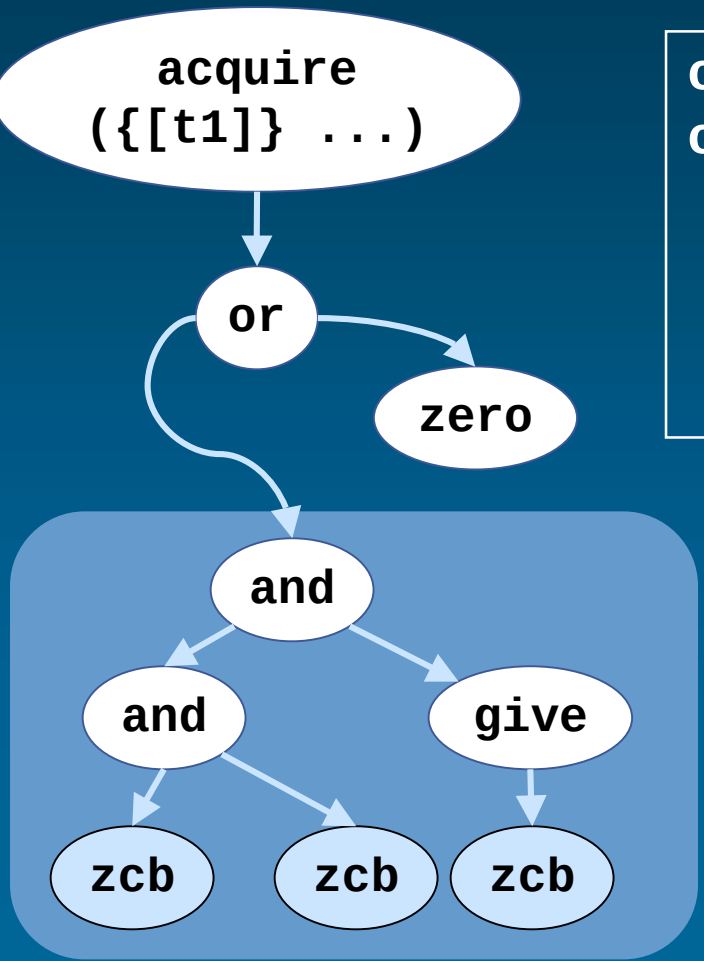
Pragmatics: The Horizon

- Each contract c has a *horizon* H (c), the latest possible date on which it may be *acquired*.
- A violation of this rule (that is, a potential acquisition after the horizon) is flagged as an *error* by the MLFi compiler
- Traps most of the typical contract design errors
- Each region r has a horizon h(r), the latest possible date on which it may be true
- The horizon of a contract is defined compositionally:

```
H(c1 +and' c2)  = min(H(c1), H(c2))
H(give c)       = H(c)
H(acquire(r, c))= h(r)
                ...
```

Reminder

Remember that the underlying contract is *arbitrary*




```
c5 : contract
c5 =  european(t1, (
                    zcb(t2,100,GBP)
          'and'     zcb(t3,100,GBP)
          'and'     give (zcb(t4,200,GBP))
          ))
```

The underlying

Observables

- Any (often uncertain) future value on which both contract parties will agree at realization date
- Observables are observed at a date
- Contracts are written “on” observables
  - e.g., deliver a cash amount at date t depending on an observed interest rate at this date

Observables (2)

- May be a number (e.g., a quoted stock price), a boolean (e.g., default, non default), or something else (e.g., a credit class)
- Observables may be manipulated in many ways to obtain new observables
  - *f(o1, o2,…) is an observable if the oi's are*
  - *an observable may be equal to an observable observed 10 days earlier*
  - *etc.*

Acquisition Date and Horizon

- The MLFi language describes what a contract *is*
- However, the *consequences for the holder* of the contract depend on the contract's *acquisition date*

- The horizon of a contract is a (compositional) property of the contract
- The acquisition date is not!

Choose When...

```
anytime : (region * contract * contract) -> contract
(* Acquire immediately the first contract and
   have the right, at any time in the argument region,
   to “give it back” against acquiring
   the second contract *)
```

Any payment (in or out) due *before* acquisition date is simply *discarded*

… and Setting the Window

Let's use here a slightly more complex region than in our previous example: the time interval, noted { [t1, t2] }, "from t1 to t2"
Note that former {[t]} is a shorthand for {[t, t]}

An American option usually comes with a pair of times:

- You cannot acquire the underlying u *before* t1;
- You cannot acquire the underlying u *after* t2;

```
anytime({[t1, t2]}, zero, u)
(* right to acquire u between t1 and t2 *)
```

American Options

```
american : (date * date * contract) -> contract
american(t1, t2, u) =
      anytime({[t1, t2]}, zero, u)
```

Zero-Coupon Again

```
one : currency -> contract
(* if you acquire the contract (one k), then
 you acquire one unit of k. *)

scale : (observable * contract) -> contract
(* if you acquire scale(o, c), then you acquire
 c, but where all incoming and outgoing payments
 are multiplied by the value of o at acquisition
 date. *)

obs_from_float : float -> observable
(* obs_from_float k is an observable always equal to k *)
```

```
zcb(t,k,cur) =
acquire({[t]},(scale(obs_from_float k, one cur)))
```

Summary So Far

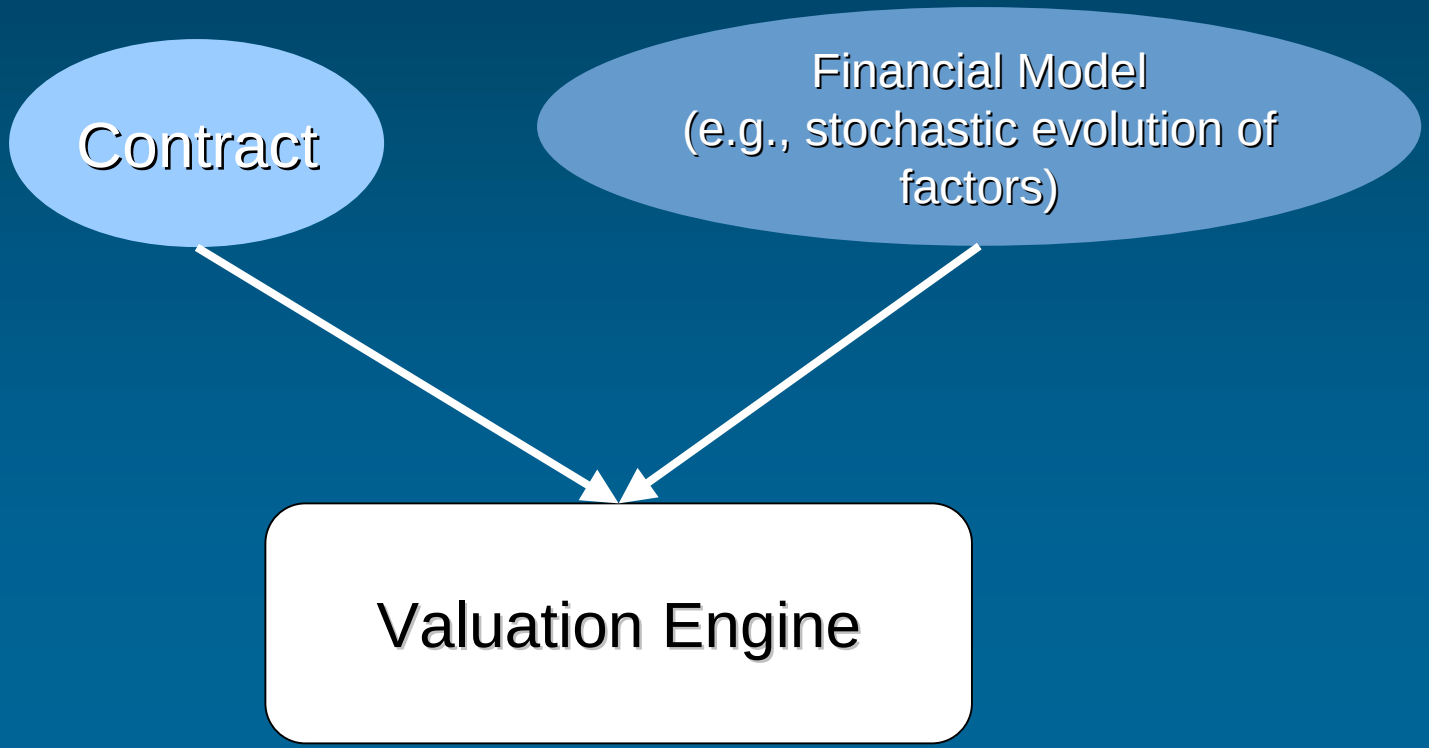
```
give     : contract -> contract
or       : (contract * contract) -> contract
and      : (contract * contract) -> contract
zero     : contract
acquire  : (region * contract) -> contract
anytime  : (region * contract * contract) -> contract
truncate : (date * contract) -> contract
scale    : (observable * contract) -> contract
…and some more besides…
```

- *Everything* is built from the combinators!
- We need an *absolutely precise* specification of what they mean

Valuation

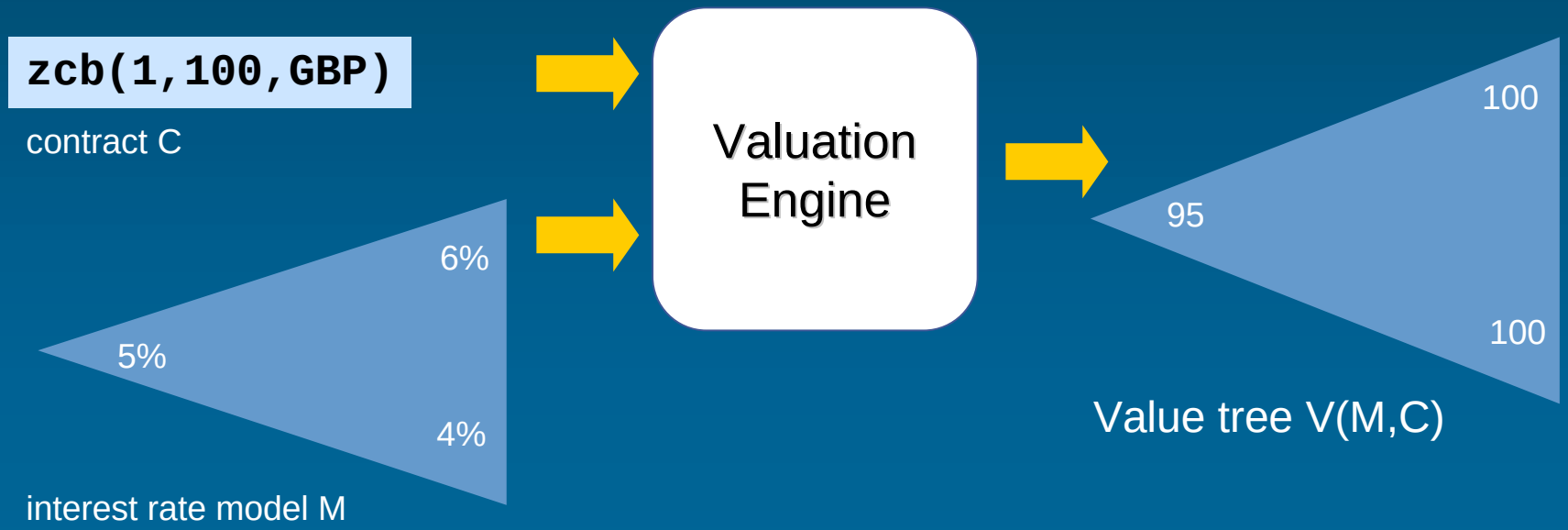
Valuation

- Once we have a precise contract specification, we may want to value it



One Possible Evaluation Model: Interest Rates "Tree"

Given a contract C, define V(M,C) to be the BDT "tree" (grid) for C under interest rate model M



Compositional Valuation

Now define V(M,C) compositionally:

V(M, **c1 and c2**) = V(M,**c1**) + V(M,**c2**)

V(M, **c1 or c2**) = max(V(M,**c1**), V(M,**c2**))

V(M, **give c**) = - V(M,**c**)

V(M, zero) = 0

...

Compositional Valuation (2)

For the combinators *anytime* and *acquire*, we must use the abstract evaluation primitives, well known from no-arbitrage pricing theory:

V(M,**`anytime(c1,c2)`**) = V(M(**`c1`**)) + snell(V(M,**`c2`**)-V(M,**`c1`**))

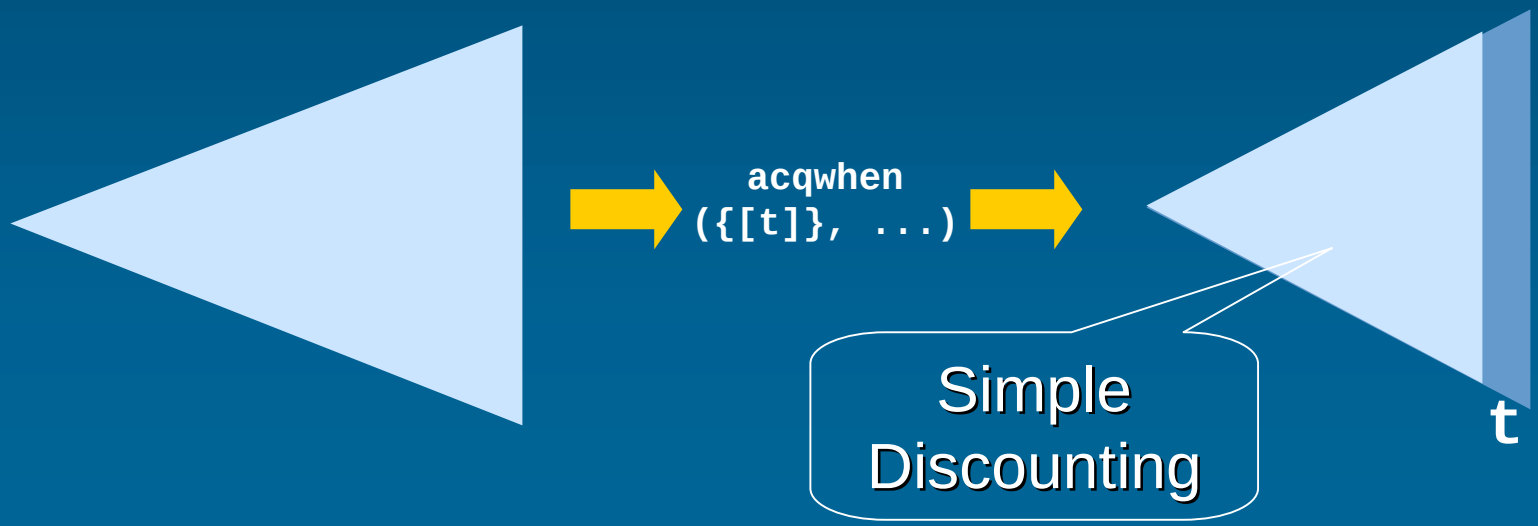
V(M, **`acquire(r, c)`**) = discount(r, V(M,**`c`**))

...

This is a major payoff! Deal with +/- 10 combinators, and we are done with valuation!

Space and Time

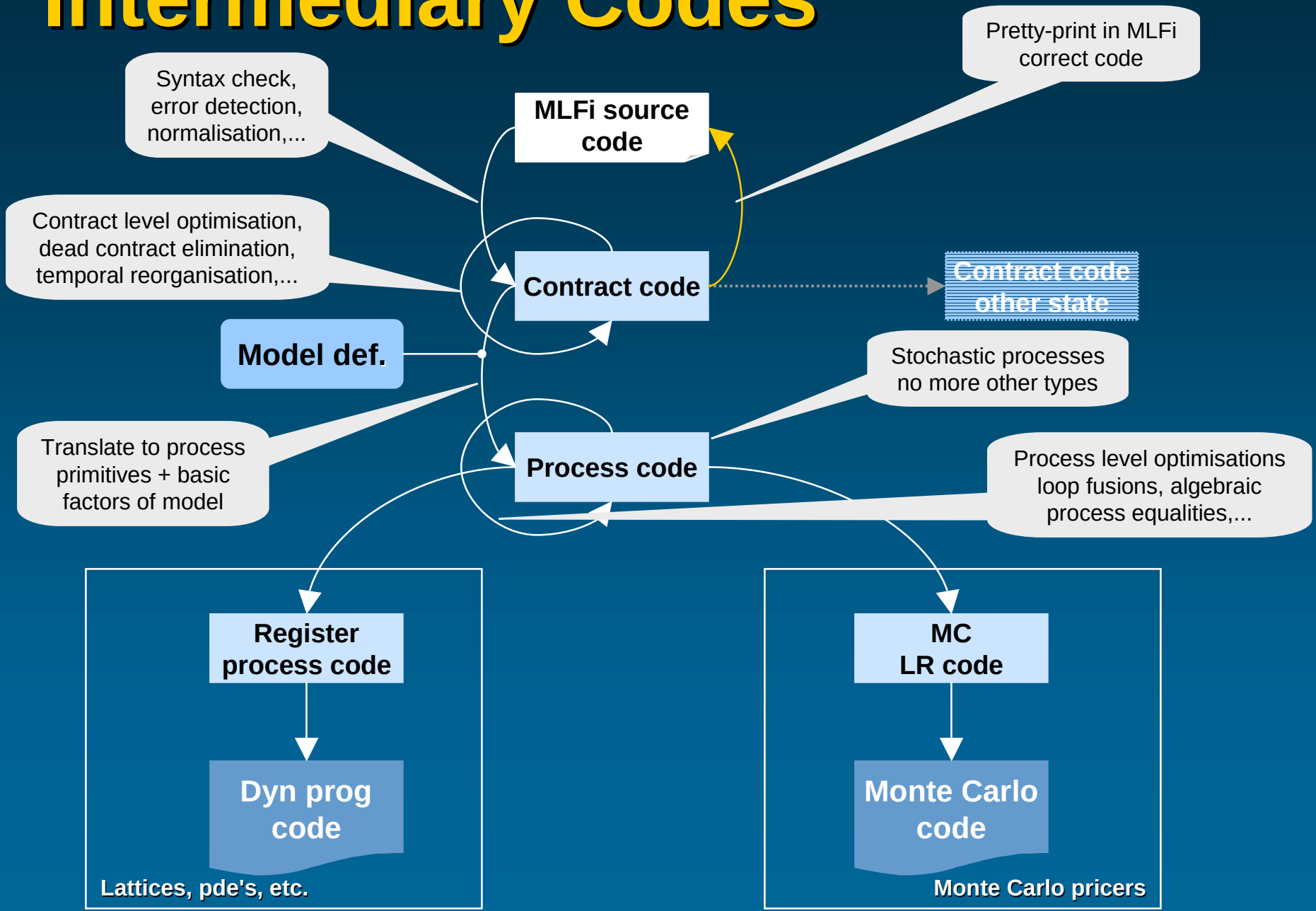
- Obvious implementation computes the value “tree” for each sub-contract
- But these value “trees” can get big
- And often, parts of them are not needed



More Precisely: What is a Model ?

- Defines its basic factors, implementation geometry
- Defines supported currencies
- Links "external" observables and contracts to "internal" pricing concepts
- Enables closed form solutions to be applied to specific (sub-) contracts
- "Accepts or refuses" to price a contract

Intermediary Codes



Optimizations

- Contracts
  - Dead contract elimination
- Processes
  - Linearity of (un)discounted evaluation
  - Linearity of change of numeraire
  - Temporal succession of discounting
  - Evaluation sharing
  - Loop fusion and closed form integration
  - Loop invariant code motion

Rescue: Compiler Technology

- Static analysis of needed calculation paths
- Data structures are computed incrementally, as they are needed (so the trees never exist in memory all at once): "slices"
- Parts that are never needed are never (or rarely) computed
- Is a deeply studied problem in theoretical computer science
- Typically: resulting code = succession of slice calculations

Slogan:

We think of the “tree” as a first class value
“all at once” (semantics) but it materializes only
“piecemeal” (implementation).

Management

Compositional Management

Managing contract c:

- “Timely monitoring of decisions that require action (e.g., own exercise decision, etc.), events to wait for (e.g., fixing, counterparty exercise decision, etc.), amounts to pay or to receive, etc.”
- We want to derive such a “management machine” exclusively from the contract’s definition

State: **<t, h, c, q, ls>**

- Acquisition date
- Manage holding up to date
- Contract definition
- Quantity
- Long (true) or short (false)

Compositional Management (2)

Now define a transition system compositionally:

<t,h,c1 and c2,q,ls>	→	**<t,h,c1,q,ls> “and” <t,h,c2,q,ls>**
<t,h,c1 or c2,q,true>	→	**<t,h,c_i,q,true>** if I choose **c_i**
<t,h,c1 or c2,q,false>	→	**<t,h,c_i,q,false>** if counterpart chooses **c_i**
<t,h,give c,q,ls>	→	**<t,h,c,q,not ls>**
<t,h,one k,q,ls>	→	**<“empty”>** receive (**ls**=true) or pay (**ls**=false) **q** units of **k**
<t,h,acquire({[t1]},c),q, ls>	→	**<t1,h,c,q,ls>** when h >= t1
<t,h,scale o c,q,ls>	→	**<t,h,c,o(t)*q,ls>**
…etc…		

Side-effect of a transition

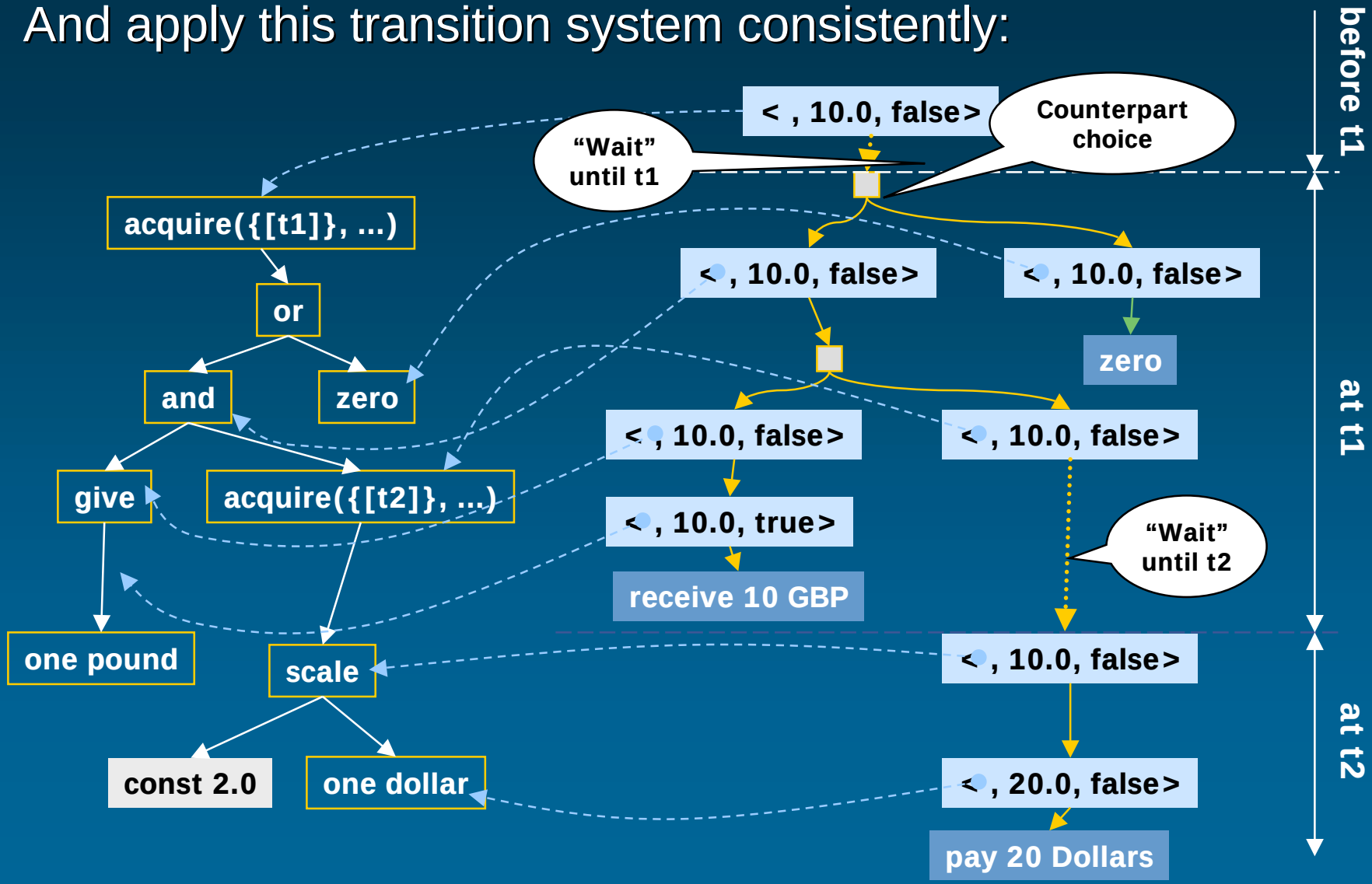
Time change

Transition condition

Decision

Compositional Management (3)

And apply this transition system consistently:



Management Automaton

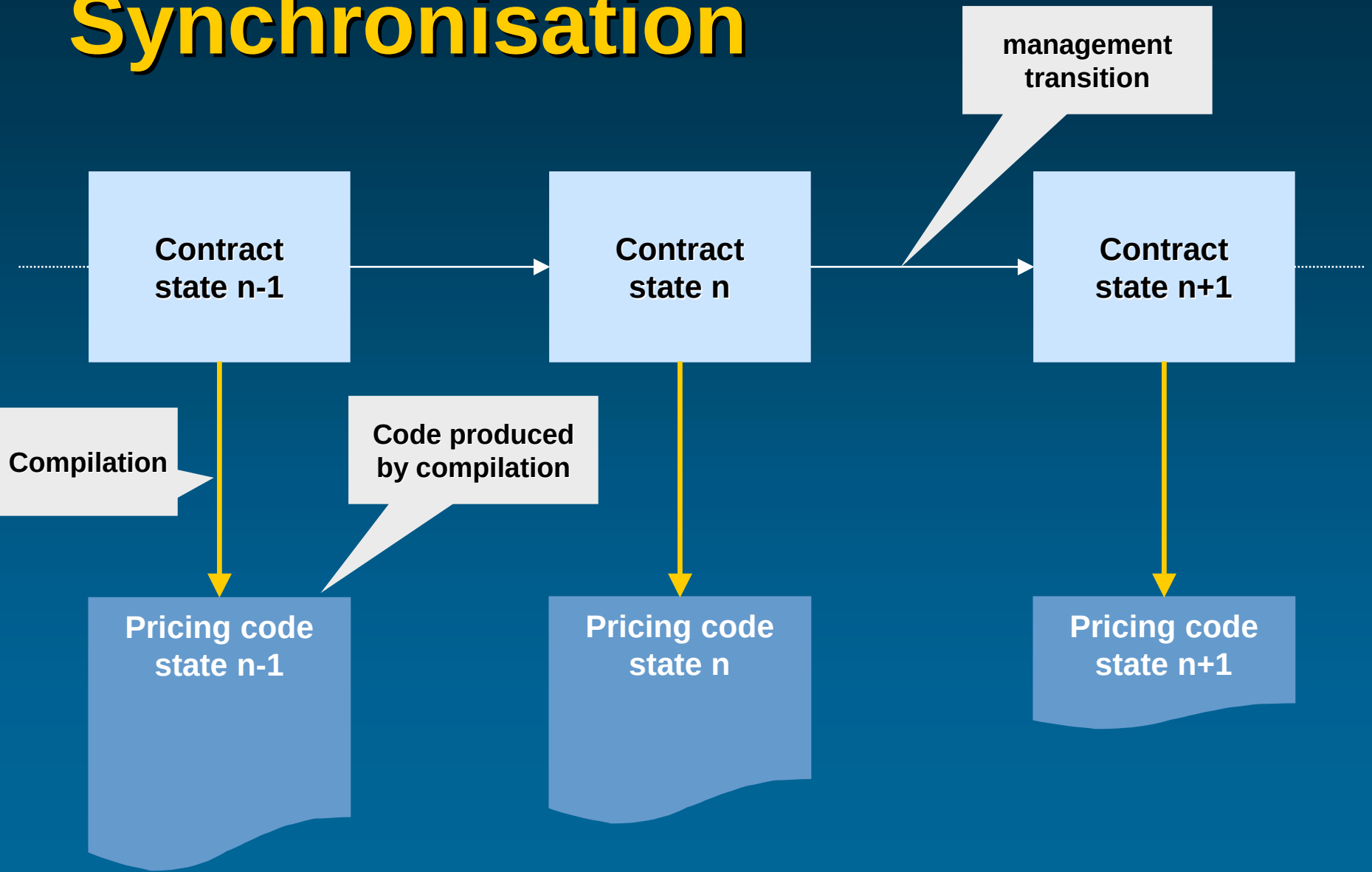
From the *description* of a contract, we derive *formally*:

- How the contract evolves through time
- How it produces side-effects (e.g., Payments)
- How it has to wait for (and react to) external signals (e.g., Exercise decisions)

Important:

For a contract in a given state, we can always deduce easily the events (signals) we are waiting for.
We accept, of course, unexpected events (bankruptcy, legal litigation, etc.).

Pricing and Management Synchronisation



Events Scheduler

- The power of a combinatorial language approach can be leveraged to develop useful and innovative applications
- Example: The Contract Events Scheduler
  - Some contract events—for instance a cash payment—may depend on one or more earlier exercise decisions or fixings
  - Although dates are often known in advance, associated payments are not, as they depend on unresolved fixings
  - We want a clear, exhaustive and compact representation of the future, to be used typically by the front or the middle office

Events Scheduler (2)

The Events Scheduler automatically populates a calendar application—Microsoft Outlook in our example—with all future events related to a given contract. No event is forgotten

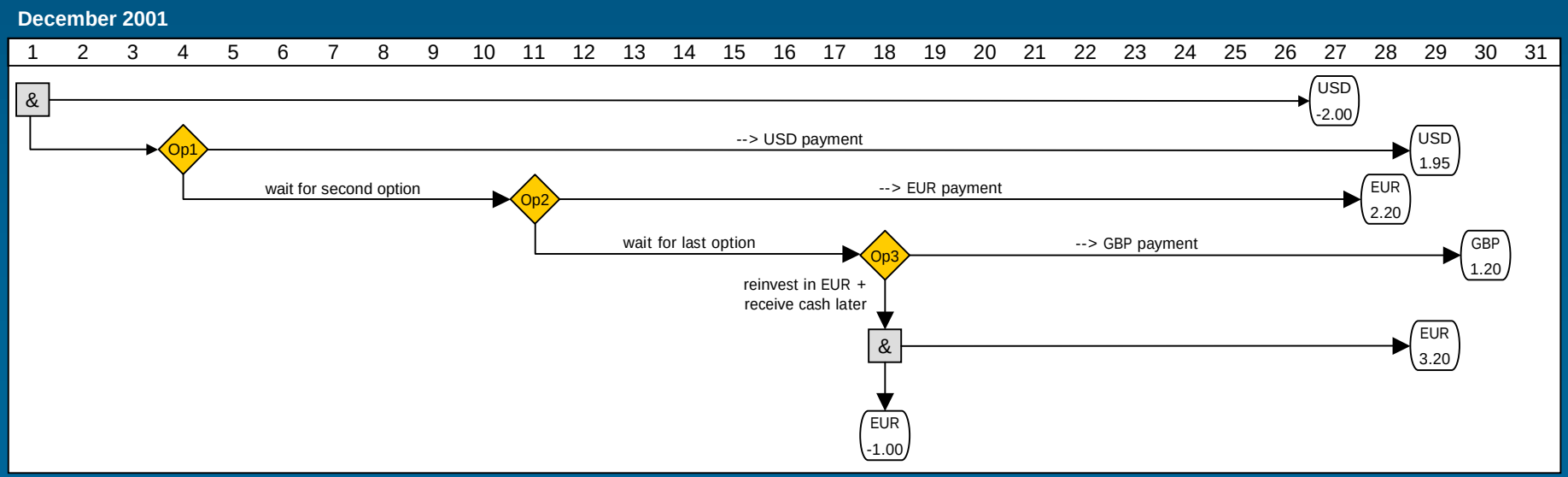
- For ease of presentation, we consider an unrealistic short-dated, one-month contract
- The contract comprises a complex 3-currency option
- The same technology may be applied to any kind of contract

The Events Scheduler reduces operational risk

The Financial Contract

Against the promise to pay USD 2.00 on December 27 (the price of the option), the holder has the right, on December 4, to choose between

- receiving USD 1.95 on December 29, or
- having the right, on December 11, to choose between
  - receiving EUR 2.20 on December 28, or
  - having the right, on December 18, to choose between
    - receiving GBP 1.20 on December 30, or
    - paying immediately one more EUR and receiving EUR 3.20 on December 29.



MLFi Contract Description

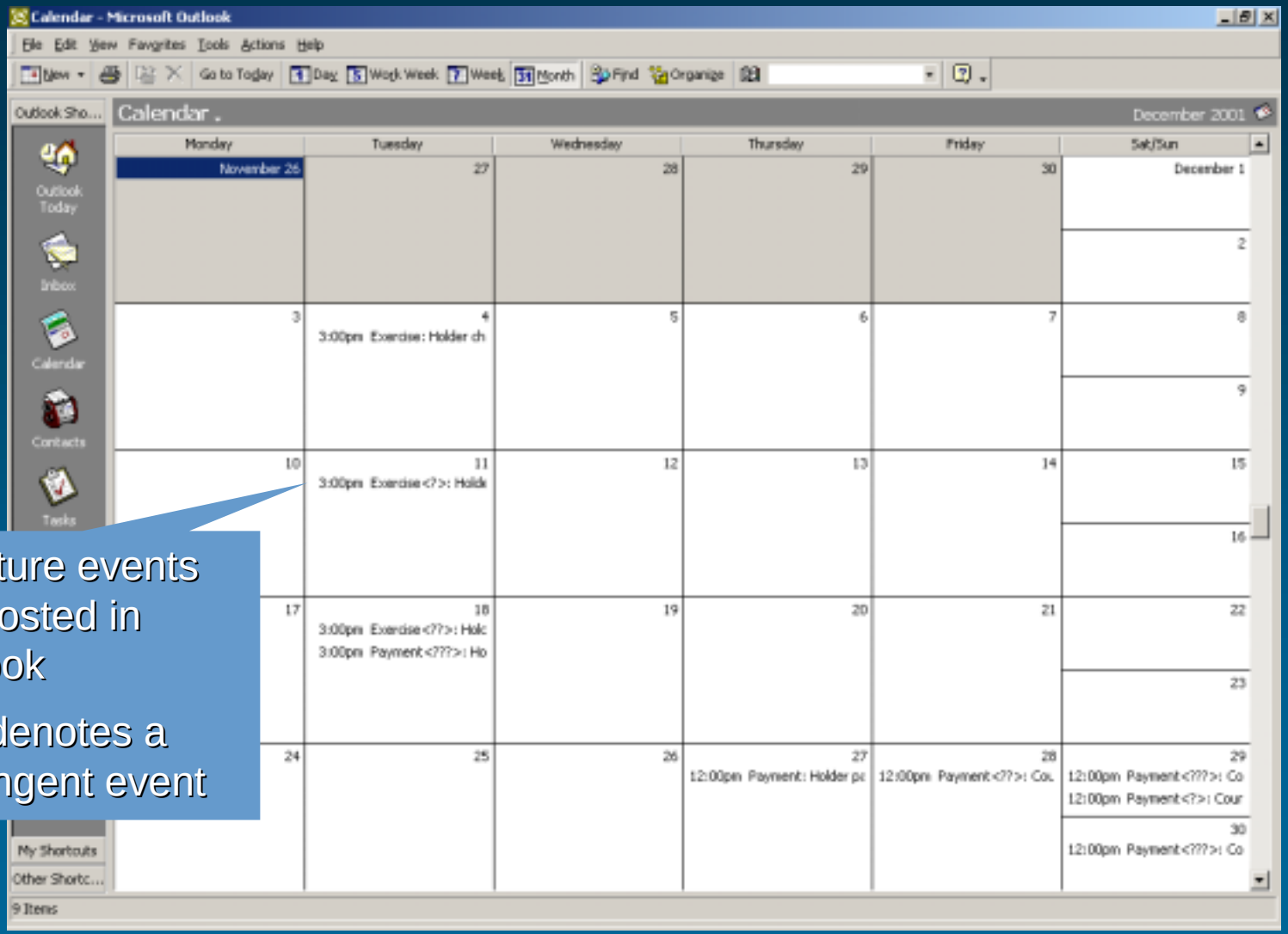
Managing and monitoring this "custom build" structure with MLFi is simple

```
let option1 =

  let strike = cashflow(USD:2.00, 2001-12-27) in

  let option2 =

    let option3 =
      let t = 2001-12-18T15:00 in
      either
        ("--> GBP payment", cashflow(GBP:1.20, 2001-12-30))
        ("reinvest in EUR + receive cash later",
         (give(cashflow(EUR:1.00, t))) 'and' cashflow(EUR:3.20, 2001-12-29))
        t in

    either
      ("--> EUR payment", cashflow(EUR:2.20, 2001-12-28))
      ("wait for last option", option3)
      2001-12-11T15:00 in

  (either
     ("--> USD payment", cashflow(USD:1.95, 2001-12-29))
     ("wait for second option", option2)
     2001-12-04T15:00) 'and' (give (strike))

let cal = calendar option1
let _ = vcalfile "test.vcs" cal
```

Contract Description

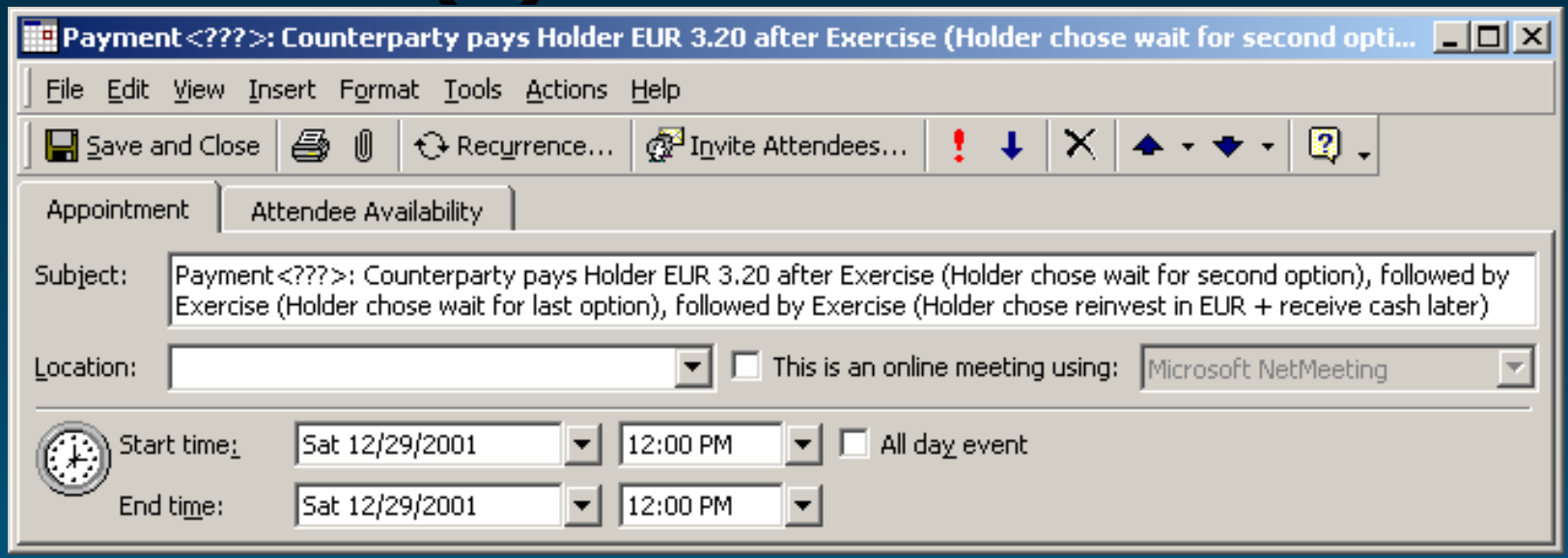
Events Scheduler

Calendar (1)



- All future events are posted in Outlook
- <?> denotes a contingent event

Calendar (2)



- <???> indicates that the payment of EUR 3.20 is contingent upon three exercise decisions
- The list of exercise decisions required for the event to occur, and precise event descriptions, are presented in chronological order:

```
… after Exercise (Holder chose wait for second
option), followed by Exercise (Holder chose wait
for last option), followed by Exercise (Holder
chose reinvest in EUR + receive cash later)
```

Much More to Be Said About MLFi

- The notion of *observables*
  - quoted prices (equities, bonds) and rates, etc.
  - default events, credit risk, etc.
- *Limit* options, *path-dependent* options
- MLFi's short and easy notation for *schedules* and operators over them
  - describe bonds, swaps, caps, etc.
  - but also multi-date exercisable optional structures
- Openness of LexiFi tools through generalized use of the XML standard

Summary

Summary

- A small set of built-in combinators
- A user-extensible library defines the zoo of contracts
- So you can define an infinite family of contracts
- Compositional (modular) algorithms for valuation (front office), management (back office) and other purposes
- A suitable formalism: *reasoning about contracts, proofs about contracts, simplifications of contracts, transforming contracts*

Summary (2)

- LexiFi develops and markets compiler technology and open applications built around the MLFi language, using the XML standard for easy interoperability
- For more information:

www.lexifi.com